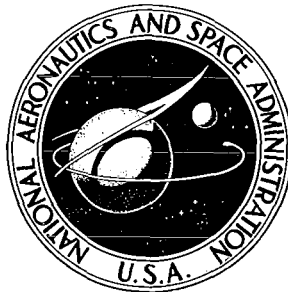


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SUPPLEMENT TO
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FLUTTER, VIBRATION, AND BUCKLING
OF TRUNCATED ORTHOTROPIC
CONICAL SHELLS WITH GENERALIZED
ELASTIC EDGE RESTRAINT

by Sidney C. Dixon and M. Latrelle Hudson
Langley Research Center
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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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INTRODUCTION

An analysis of the flutter, vibration, and buckling of orthotropic conical shells is presented in NASA TN D-5759. Certain lengthy expressions which were developed in the analysis are required for complete documentation but were considered to be of interest to only a limited group of readers. This supplement to NASA TN D-5759 presents these expressions in four sections. The first section gives the details of the solution of the compatibility equations. The second section presents the expressions l_{ij} , g_{ij} , h_{ij} , and \bar{e}_{ij} that appear in the boundary conditions for the stress functions. The third section presents expressions for the matrix elements b_{ij} , c_{ij} , d_{ij} , e_{ij} , and f_{ij} of the governing stability equation (eq. (8)). The fourth section presents expressions for the nondimensional parameters S_{ij} and M_{ij} which relate the stiffness and inertia characteristics of end rings to the ring and shell material properties and geometry. The parameters S_{ij} and M_{ij} appear in the expressions presented in the second and third sections. The notation form $S_{ij,1}$ refers to the small end of the conical shell and $S_{ij,2}$ to the large end. All symbols not defined herein are defined in the list of symbols in NASA TN D-5759.

SOLUTION OF COMPATIBILITY EQUATION

The compatibility equation is

$$\left(y \epsilon_{y\theta B, \phi} \right)_{,y} - \epsilon_{yB, \phi \phi} - \left(y^2 \epsilon_{\theta B, y} \right)_{,y} + y \epsilon_{yB, y} = y \cot \alpha w_{B, yy} \quad (S1)$$

or, in terms of the stress function F ,

$$L_2(F) = -B_\theta \left(1 - \mu'_\theta \mu'_y \right) w_{B, yy} \cot \alpha \quad (S2)$$

where

$$\begin{aligned} L_2(\) = y(\), yyy + \frac{B_\theta}{By} \left[\frac{1}{y^3}(\), \phi \phi \phi \phi + \frac{2}{y^3}(\), \phi \phi + \frac{1}{y^2}(\), y - \frac{1}{y}(\), yy \right] \\ + \left[\frac{B_\theta}{By\theta} \left(1 - \mu'_\theta \mu'_y \right) - 2\mu'_\theta \right] \left[\frac{1}{y^3}(\), \phi \phi - \frac{1}{y^2}(\), y \phi \phi + \frac{1}{y}(\), yy \phi \phi \right] \end{aligned} \quad (S3)$$

Equation (S2) has variable coefficients that reduce to constant coefficients by use of the transformation of coordinates

$$y = y_1 e^x \quad (S4)$$

For simple harmonic motion the displacement w_B is assumed to be

$$w_B = w_B^*(x) \cos n\phi e^{i\omega t} \quad (S5)$$

where

$$w_B^*(x) = a_0 + \bar{a}_0 + \sum_{m=1}^M a_m \sin \frac{m\pi x}{x_1} \quad (S6)$$

If the stress function F is taken to be

$$F = e^x f(x) \cos n\phi e^{i\omega t} \quad (S7)$$

and equations (S4) and (S6) are utilized, equation (S2) reduces to the ordinary differential equation

$$f_{,xxxx} + P_1 f_{,xx} + P_2 f = -B_\theta \left(1 - \mu'_\theta \mu'_y \right) y_1 \cos \alpha \left(w_{B, xx}^* - w_{B, x}^* \right) \quad (S8)$$

where

$$\left. \begin{aligned} P_1 &= -\left(1 + \frac{B_\theta}{B_y}\right) - n^2 \left[\frac{B_\theta}{B_y \theta} \left(1 - \mu'_\theta \mu'_y\right) - 2\mu'_\theta \right] \\ P_2 &= \frac{B_\theta}{B_y} (n^2 - 1)^2 \end{aligned} \right\} \quad (S9)$$

By the use of equation (S7), equation (S8) contains only even derivatives of f with respect to x . The homogeneous solution is readily found to be

$$f_h = \bar{A}_1 e^{\lambda_1 x} + \bar{A}_2 e^{-\lambda_1 x} + \bar{A}_3 e^{\lambda_2 x} + \bar{A}_4 e^{-\lambda_2 x} \quad (S10)$$

where

$$\left. \begin{aligned} \lambda_1 &= \left[\frac{-P_1 + (P_1^2 - 4P_2)^{1/2}}{2} \right]^{1/2} \\ \lambda_2 &= \left[\frac{-P_1 - (P_1^2 - 4P_2)^{1/2}}{2} \right]^{1/2} \end{aligned} \right\} \quad (S11)$$

The particular solution is obtained by substituting

$$f_p = d_0 + \sum_{m=1}^M \left(d_m \cos \frac{m\pi x}{x_1} + e_m \sin \frac{m\pi x}{x_1} \right) \quad (S12)$$

into equation (S8) and matching coefficients; this gives

$$\left. \begin{aligned} d_0 &= \frac{B_\theta (1 - \mu'_\theta \mu'_y) y_1 \cot \alpha}{P_2} \bar{a}_0 \\ d_m &= \frac{B_\theta (1 - \mu'_\theta \mu'_y) y_1 \cot \alpha}{\left(\frac{m\pi}{x_1}\right)^4 - P_1 \left(\frac{m\pi}{x_1}\right)^2 + P_2} \frac{m\pi}{x_1} a_m \\ e_m &= \frac{B_\theta (1 - \mu'_\theta \mu'_y) y_1 \cot \alpha}{\left(\frac{m\pi}{x_1}\right)^4 - P_1 \left(\frac{m\pi}{x_1}\right)^2 + P_2} \left(\frac{m\pi}{x_1}\right)^2 a_m \end{aligned} \right\} \quad (S13)$$

Thus $f = f_h + f_p$ becomes

$$f = B_\theta \left(1 - \mu'_\theta \mu'_y\right) y_1 \cot \alpha \left\{ A_1 e^{\lambda_1 x} + A_2 e^{-\lambda_1 x} + A_3 e^{\lambda_2 x} + A_4 e^{-\lambda_2 x} + \frac{\bar{a}_0}{P_2} \right. \\ \left. + \sum_{m=1}^M \frac{\left[\left(\frac{m\pi}{x_1}\right)^2 \sin \frac{m\pi x}{x_1} + \frac{m\pi}{x_1} \cos \frac{m\pi x}{x_1} \right]}{\left(\frac{m\pi}{x_1}\right)^4 - P_1 \left(\frac{m\pi}{x_1}\right)^2 + P_2} a_m \right\} \quad (S14)$$

The stress resultants become

$$N_{\theta B} = \frac{B_\theta \left(1 - \mu'_\theta \mu'_y\right) \cot \alpha}{y_1 e^x} \left\{ A_1 \lambda_1 (1 + \lambda_1) e^{\lambda_1 x} - A_2 \lambda_1 (1 - \lambda_1) e^{-\lambda_1 x} + A_3 \lambda_2 (1 + \lambda_2) e^{\lambda_2 x} \right. \\ \left. - A_4 \lambda_2 (1 - \lambda_2) e^{-\lambda_2 x} - \sum_{m=1}^M \frac{\left[\left(\frac{m\pi}{x_1}\right)^4 + \left(\frac{m\pi}{x_1}\right)^2 \right]}{\left(\frac{m\pi}{x_1}\right)^4 - P_1 \left(\frac{m\pi}{x_1}\right)^2 + P_2} \sin \frac{m\pi x}{x_1} a_m \right\} \cos n\phi \quad (S15)$$

$$N_{yB} = \frac{B_\theta \left(1 - \mu'_\theta \mu'_y\right) \cot \alpha}{y_1 e^x} \left(A_1 (1 + \lambda_1 - n^2) e^{\lambda_1 x} + A_2 (1 - \lambda_1 - n^2) e^{-\lambda_1 x} \right. \\ \left. + A_3 (1 + \lambda_2 - n^2) e^{\lambda_2 x} + A_4 (1 - \lambda_2 - n^2) e^{-\lambda_2 x} + \frac{\bar{a}_0 (1 - n^2)}{P_2} \right. \\ \left. + \sum_{m=1}^M \frac{\left\{ \left[\left(\frac{m\pi}{x_1}\right)^3 + \frac{m\pi}{x_1} (1 - n^2) \right] \cos \frac{m\pi x}{x_1} - n^2 \left(\frac{m\pi}{x_1}\right)^2 \sin \frac{m\pi x}{x_1} \right\} a_m}{\left(\frac{m\pi}{x_1}\right)^4 - P_1 \left(\frac{m\pi}{x_1}\right)^2 + P_2} \right) \cos n\phi \quad (S16)$$

and

$$N_{y\theta B} = \frac{nB_\theta(1 - \mu'_\theta\mu'_y)\cot\alpha}{y_1e^x} \left\{ A_1\lambda_1e^{\lambda_1x} - A_2\lambda_1e^{-\lambda_1x} + A_3\lambda_2e^{\lambda_2x} - A_4\lambda_2e^{-\lambda_2x} \right. \\ \left. + \sum_{m=1}^M \frac{\left[\left(\frac{m\pi}{x_1}\right)^3 \cos \frac{m\pi x}{x_1} - \left(\frac{m\pi}{x_1}\right) \sin \frac{m\pi x}{x_1}\right]}{\left(\frac{m\pi}{x_1}\right)^4 - P_1\left(\frac{m\pi}{x_1}\right)^2 + P_2} a_m \right\} \sin n\phi \quad (S17)$$

The constitutive equations are

$$\epsilon_{\theta B} = \frac{1}{B_\theta(1 - \mu'_\theta\mu'_y)}(N_{\theta B} - \mu'_\theta N_{yB}) \quad (S18a)$$

$$\epsilon_{yB} = \frac{1}{B_\theta(1 - \mu'_\theta\mu'_y)}(N_{yB} - \mu'_y N_{\theta B}) \quad (S18b)$$

$$\epsilon_{y\theta B} = \frac{1}{B_{y\theta}} N_{y\theta B} \quad (S18c)$$

and the linear strain-displacement relations are

$$\epsilon_{yB} = \frac{1}{y_1e^x} u_{B,x} \quad (S19a)$$

$$\epsilon_{\theta B} = \frac{1}{y_1e^x} (u_B - w_B \cot\alpha + v_{B,\phi}) \quad (S19b)$$

$$\epsilon_{y\theta B} = \frac{1}{y_1e^x} (v_{B,x} - v_B + u_{B,\phi}) \quad (S19c)$$

Substituting equations (S16), (S17), (S19a), and (S19b) into equation (S18b) and integrating give

$$\begin{aligned}
u_B = & \left\{ \frac{B_\theta}{B_y} \cot \alpha \left[A_1 \psi_1 e^{\lambda_1 x} + A_2 \psi_2 e^{-\lambda_1 x} + A_3 \psi_3 e^{\lambda_2 x} + A_4 \psi_4 e^{-\lambda_2 x} - \frac{\bar{a}_0}{P_2} (n^2 - 1) x \right. \right. \\
& + \sum_{m=1}^M \frac{a_m}{\left(\frac{m\pi}{x_1} \right)^4 - P_1 \left(\frac{m\pi}{x_1} \right)^2 + P_2} \left(\left[\left(\frac{m\pi}{x_1} \right)^2 + 1 - n^2 \right] \sin \frac{m\pi x}{x_1} - \left\{ \mu'_y \left[\left(\frac{m\pi}{x_1} \right)^3 + \frac{m\pi}{x_1} \right] \right. \right. \\
& \left. \left. \left. - \frac{m\pi}{x_1} n^2 \right\} \cos \frac{m\pi x}{x_1} \right) \right] + C_1 \left. \right\} \cos n\phi \quad (S20)
\end{aligned}$$

where

$$\left. \begin{aligned}
\psi_1 &= \frac{1 + \lambda_1 - n^2}{\lambda_1} - \mu'_y (1 + \lambda_1) \\
\psi_2 &= \frac{\lambda_1 + n^2 - 1}{\lambda_1} - \mu'_y (1 - \lambda_1) \\
\psi_3 &= \frac{1 + \lambda_2 - n^2}{\lambda_2} - \mu'_y (1 + \lambda_2) \\
\psi_4 &= \frac{\lambda_2 + n^2 - 1}{\lambda_2} - \mu'_y (1 - \lambda_2)
\end{aligned} \right\} \quad (S21)$$

Substituting equation (S17) and equation (S19c) into equation (S18c), utilizing equation (S20), and integrating give

$$\begin{aligned}
v_B = & n \cot \alpha \left[A_1 \Lambda_1 e^{\lambda_1 x} + A_2 \Lambda_2 e^{-\lambda_1 x} + A_3 \Lambda_3 e^{\lambda_2 x} + A_4 \Lambda_4 e^{-\lambda_2 x} + \frac{1+x}{n^2-1} \bar{a}_0 - \frac{C_1}{\cot \alpha} + \frac{C_2 e^x}{\cot \alpha} \right. \\
& + \sum_{m=1}^M \frac{a_m}{\left(\frac{m\pi}{x_1} \right)^4 - P_1 \left(\frac{m\pi}{x_1} \right)^2 + P_2} \left(\left\langle \left(\frac{m\pi}{x_1} \right)^2 \left[\frac{B_\theta}{B_y \theta} (1 - \mu'_\theta \mu'_y) - \mu'_\theta \right] + \frac{B_\theta}{B_y} (n^2 - 1) \right\rangle \sin \frac{m\pi x}{x_1} \right. \\
& \left. \left. - \frac{m\pi}{x_1} (1 - \mu'_y) \frac{B_\theta}{B_y} \cos \frac{m\pi x}{x_1} \right) \right] \sin n\phi \quad (S22)
\end{aligned}$$

where

$$\left. \begin{aligned} \Lambda_1 &= \frac{(B_\theta/B_{y\theta})(1 - \mu'_\theta \mu'_y)\lambda_1 + (B_\theta/B_y)\psi_1}{\lambda_1 - 1} \\ \Lambda_2 &= \frac{(B_\theta/B_{y\theta})(1 - \mu'_\theta \mu'_y)\lambda_1 - (B_\theta/B_y)\psi_2}{\lambda_1 + 1} \\ \Lambda_3 &= \frac{(B_\theta/B_{y\theta})(1 - \mu'_\theta \mu'_y)\lambda_2 + (B_\theta/B_y)\psi_3}{\lambda_2 - 1} \\ \Lambda_4 &= \frac{(B_\theta/B_{y\theta})(1 - \mu'_\theta \mu'_y)\lambda_2 - (B_\theta/B_y)\psi_4}{\lambda_2 + 1} \end{aligned} \right\} \quad (S23)$$

The constants of integration C_1 and C_2 are determined by substituting the expressions for u_B , v_B , w_B , $N_{\theta B}$, and N_{yB} into equation (S18a) and matching appropriate coefficients; this gives

$$\left. \begin{aligned} C_1 &= \cot \alpha \left[\frac{\bar{a}_0}{P_2} \left(n^2 \frac{B_\theta}{B_y} - \mu'_\theta \right) - \frac{a_0}{n^2 - 1} \right] \\ C_2 &= 0 \end{aligned} \right\} \quad (S24)$$

The constants of integration A_1 to A_4 are determined from the boundary condition for u_B and v_B , the first two of equations (B47). When the expressions for u_B , v_B , w_B , N_{yB} , and $N_{y\theta B}$ are substituted into equations (B47), the boundary conditions become

$$\begin{bmatrix} l_{11} & l_{12} & l_{13} & l_{14} \\ l_{21} & l_{22} & l_{23} & l_{24} \\ l_{31} & l_{32} & l_{33} & l_{34} \\ l_{41} & l_{42} & l_{43} & l_{44} \end{bmatrix} \begin{Bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{Bmatrix} = \begin{bmatrix} g_{11} & g_{12} & g_{13} & \dots & g_{1j} \\ g_{21} & g_{22} & g_{23} & \dots & g_{2j} \\ g_{31} & g_{32} & g_{33} & \dots & g_{3j} \\ g_{41} & g_{42} & g_{43} & \dots & g_{4j} \end{bmatrix} \begin{Bmatrix} a_0 \\ \bar{a}_0 \\ a_1 \\ \vdots \\ a_M \end{Bmatrix} \quad (j = M + 2) \quad (S25)$$

where l_{ij} and g_{ij} are given in the next section.

Equation (S25) can be solved for A_1 to A_4 in the form

$$A_i = \Omega_i a_o + \Gamma_i \bar{a}_o + \sum_{m=1}^M \Phi_{im} a_m \quad (i = 1, 2, 3, 4) \quad (S26)$$

With A_i known in terms of a_o , \bar{a}_o , and a_m , then F , u_B , and v_B can be expressed in terms of these coefficients as

$$F = \cot \alpha B_\theta \left(1 - \mu'_\theta \mu'_y\right) y_1 e^x \left\{ a_o \left(\Omega_1 e^{\lambda_1 x} + \Omega_2 e^{-\lambda_1 x} + \Omega_3 e^{\lambda_2 x} + \Omega_4 e^{-\lambda_2 x} \right) + \bar{a}_o \left(\Gamma_1 e^{\lambda_1 x} + \Gamma_2 e^{-\lambda_1 x} + \Gamma_3 e^{\lambda_2 x} + \Gamma_4 e^{-\lambda_2 x} + \frac{1}{P_2} \right) + \sum_{m=1}^M a_m \left[\Phi_{1m} e^{\lambda_1 x} + \Phi_{2m} e^{-\lambda_1 x} + \Phi_{3m} e^{\lambda_2 x} + \Phi_{4m} e^{-\lambda_2 x} + \frac{\left(\frac{m\pi}{x_1}\right)^2 \sin \frac{m\pi x}{x_1} + \frac{m\pi}{x_1} \cos \frac{m\pi x}{x_1}}{\left(\frac{m\pi}{x_1}\right)^4 - P_1 \left(\frac{m\pi}{x_1}\right)^2 + P_2} \right] \right\} \cos n\phi \quad (S27)$$

$$u_B = \cot \alpha (B_\theta/B_y) \left\{ a_o \left[\Omega_1 \psi_1 e^{\lambda_1 x} + \Omega_2 \psi_2 e^{-\lambda_1 x} + \Omega_3 \psi_3 e^{\lambda_2 x} + \Omega_4 \psi_4 e^{-\lambda_2 x} + \frac{1}{(B_\theta/B_y)(1-n^2)} \right] + \bar{a}_o \left[\Gamma_1 \psi_1 e^{\lambda_1 x} + \Gamma_2 \psi_2 e^{-\lambda_1 x} + \Gamma_3 \psi_3 e^{\lambda_2 x} + \Gamma_4 \psi_4 e^{-\lambda_2 x} - \frac{x}{(B_\theta/B_y)(n^2-1)} + \frac{n^2 - \mu'_y}{P_2} \right] + \sum_{m=1}^M a_m \left[\Phi_{1m} \psi_1 e^{\lambda_1 x} + \Phi_{2m} \psi_2 e^{-\lambda_1 x} + \Phi_{3m} \psi_3 e^{\lambda_2 x} + \Phi_{4m} \psi_4 e^{-\lambda_2 x} + \frac{1}{\left(\frac{m\pi}{x_1}\right)^4 - P_1 \left(\frac{m\pi}{x_1}\right)^2 + P_2} \left(\left[\left(\frac{m\pi}{x_1}\right)^2 + 1 - n^2 \right] \sin \frac{m\pi x}{x_1} - \left\{ \mu'_y \left[\left(\frac{m\pi}{x_1}\right)^3 + \frac{m\pi}{x_1} \right] - n^2 \frac{m\pi}{x_1} \right\} \cos \frac{m\pi x}{x_1} \right) \right] \right\} \cos n\phi \quad (S28)$$

and

$$\begin{aligned}
v_B = n \cot \alpha & \left\{ a_o \left(\Lambda_1 \Omega_1 e^{\lambda_1 x} + \Lambda_2 \Omega_2 e^{-\lambda_1 x} + \Lambda_3 \Omega_3 e^{\lambda_2 x} + \Lambda_4 \Omega_4 e^{-\lambda_2 x} + \frac{1}{n^2 - 1} \right) + \bar{a}_o \left[\Lambda_1 \Gamma_1 e^{\lambda_1 x} \right. \right. \\
& + \Lambda_2 \Gamma_2 e^{-\lambda_1 x} + \Lambda_3 \Gamma_3 e^{\lambda_2 x} + \Lambda_4 \Gamma_4 e^{-\lambda_2 x} + \frac{x}{n^2 - 1} + \frac{\mu'_y - 1}{(n^2 - 1)^2} \left. \right] + \sum_{m=1}^M a_m \left[\Lambda_1 \Phi_{1m} e^{\lambda_1 x} \right. \\
& + \Lambda_2 \Phi_{2m} e^{-\lambda_1 x} + \Lambda_3 \Phi_{3m} e^{\lambda_2 x} + \Lambda_4 \Phi_{4m} e^{-\lambda_2 x} + \frac{1}{\left(\frac{m\pi}{x_1} \right)^4 - P_1 \left(\frac{m\pi}{x_1} \right)^2 + P_2} \left(\left(\frac{m\pi}{x_1} \right) \left[\frac{B_y \theta}{B_y} \left(1 \right. \right. \right. \\
& \left. \left. \left. - \mu'_\theta \mu'_y \right) - \mu'_\theta \right] + \frac{B_\theta}{B_y} (n^2 - 1) \right) \sin \frac{m\pi x}{x_1} - \frac{m\pi}{x_1} \frac{B_\theta}{B_y} (1 - \mu'_y) \cos \frac{m\pi x}{x_1} \right) \left. \right] \left. \right\} \sin n\phi \quad (S29)
\end{aligned}$$

For symmetric deformations the transformed compatibility equation is

$$F_{O,xx} - \frac{B_\theta}{B_y} F_O = \cot \alpha B_\theta (1 - \mu'_\theta \mu'_y) w_{B,x}^* \quad (S30)$$

and the solution is readily found to be

$$F_O = B_\theta (1 - \mu'_\theta \mu'_y) \cot \alpha \left[B_1 e^{\lambda_o x} + B_2 e^{-\lambda_o x} + \frac{\bar{a}_o}{B_\theta/B_y} + \sum_{m=1}^M \frac{\frac{m\pi}{x_1} \cos \frac{m\pi}{x_1}}{\left(\frac{m\pi}{x_1} \right)^2 + (B_\theta/B_y)} a_m \right] \quad (S31)$$

where

$$\lambda_o = \sqrt{B_\theta/B_y}$$

The displacement $v_{oB} = 0$ and u_{oB} is given by

$$u_{oB} = \cot \alpha \left[B_1 (\lambda_o - \mu'_\theta) e^{\lambda_o x} - B_2 (\lambda_o + \mu'_\theta) e^{-\lambda_o x} \right. \\ \left. + \sum_{m=1}^M \frac{(B_\theta/B_y) \left(\sin \frac{m\pi x}{x_1} - \mu'_y \frac{m\pi}{x_1} \cos \frac{m\pi x}{x_1} \right) a_m}{(m\pi/x_1)^2 + (B_\theta/B_y)} + a_o + \bar{a}_o \left(x - \frac{\mu'_\theta}{B_\theta/B_y} \right) \right] \quad (S32)$$

The boundary conditions become

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{Bmatrix} B_1 \\ B_2 \end{Bmatrix} = \begin{bmatrix} \bar{e}_{11} & \bar{e}_{12} & \bar{e}_{13} & \dots & \bar{e}_{1j} \\ \bar{e}_{21} & \bar{e}_{22} & \bar{e}_{23} & \dots & \bar{e}_{2j} \end{bmatrix} \begin{Bmatrix} a_o \\ \bar{a}_o \\ a_1 \\ \vdots \\ a_M \end{Bmatrix} \quad (j = M + 2) \quad (S33)$$

where h_{ij} and \bar{e}_{ij} are given in the next section. Equation (S33) can be solved for B_1 and B_2 in the form

$$B_i = \Omega_{oi} + \Gamma_{oi} \bar{a}_o + \sum_{m=1}^M \Phi_{oim} a_m \quad (i = 1, 2) \quad (S34)$$

With B_i known in terms of a_o , \bar{a}_o , and a_m , then F_o and u_{oB} can be expressed in terms of these coefficients as

$$F_o = B_\theta (1 - \mu'_\theta \mu'_y) \cot \alpha \left\{ a_o \left(\Omega_{o1} e^{\lambda_o x} + \Omega_{o2} e^{-\lambda_o x} \right) + \bar{a}_o \left(\Gamma_{o1} e^{\lambda_o x} + \Gamma_{o2} e^{-\lambda_o x} + \frac{1}{B_\theta/B_y} \right) \right. \\ \left. + \sum_{m=1}^M a_m \left[\Phi_{o1m} e^{\lambda_o x} + \Phi_{o2m} e^{-\lambda_o x} + \frac{(m\pi/x_1) \cos (m\pi x/x_1)}{(m\pi/x_1)^2 + (B_\theta/B_y)} \right] \right\} \quad (S35)$$

$$\begin{aligned}
u_{oB} = \cot \alpha & \left\{ a_o \left[\Omega_{o1} \left(\sqrt{B_\theta/B_y} - \mu'_\theta \right) e^{\lambda_o x} - \Omega_{o2} \left(\sqrt{B_\theta/B_y} + \mu'_\theta \right) e^{-\lambda_o x} + 1 \right] \right. \\
& + \tilde{a}_o \left[\Gamma_{o1} \left(\sqrt{B_\theta/B_y} - \mu'_\theta \right) e^{\lambda_o x} - \Gamma_{o2} \left(\sqrt{B_\theta/B_y} + \mu'_\theta \right) e^{-\lambda_o x} + x - \frac{\mu'_\theta}{B_\theta/B_y} \right] \\
& + \sum_{m=1}^M a_m \left[\Phi_{o1m} \left(\sqrt{B_\theta/B_y} - \mu'_\theta \right) e^{\lambda_o x} - \Phi_{o2m} \left(\sqrt{B_\theta/B_y} + \mu'_\theta \right) e^{-\lambda_o x} \right. \\
& \left. \left. + \frac{B_\theta/B_y}{(m\pi/x_1)^2 + (B_\theta/B_y)} \left(\sin \frac{m\pi x}{x_1} - \frac{B_y}{B_\theta} \mu'_\theta \frac{m\pi}{x_1} \cos \frac{m\pi x}{x_1} \right) \right] \right\} \quad (S36)
\end{aligned}$$

EXPRESSIONS FOR l_{ij} , g_{ij} , h_{ij} , AND \bar{e}_{ij}

The expressions for l_{ij} are

$$\begin{aligned}
 l_{11} &= \left(1 - \frac{\mu_{\theta}'^2}{B_{\theta}/B_y}\right) (1 + \lambda_1 - n^2) - \cot \alpha \frac{B_{\theta}}{B_y} \frac{Q_{11,1}}{Z} \psi_1 - n \cot \alpha \frac{Q_{12,1}}{Z} \Lambda_1 \\
 l_{12} &= \left(1 - \frac{\mu_{\theta}'^2}{B_{\theta}/B_y}\right) (1 - \lambda_1 - n^2) - \cot \alpha \frac{B_{\theta}}{B_y} \frac{Q_{11,1}}{Z} \psi_2 - n \cot \alpha \frac{Q_{12,1}}{Z} \Lambda_2 \\
 l_{13} &= \left(1 - \frac{\mu_{\theta}'^2}{B_{\theta}/B_y}\right) (1 + \lambda_2 - n^2) - \cot \alpha \frac{B_{\theta}}{B_y} \frac{Q_{11,1}}{Z} \psi_3 - n \cot \alpha \frac{Q_{12,1}}{Z} \Lambda_3 \\
 l_{14} &= \left(1 - \frac{\mu_{\theta}'^2}{B_{\theta}/B_y}\right) (1 - \lambda_2 - n^2) - \cot \alpha \frac{B_{\theta}}{B_y} \frac{Q_{11,1}}{Z} \psi_4 - n \cot \alpha \frac{Q_{12,1}}{Z} \Lambda_4 \\
 l_{21} &= \left(\frac{R_2}{R_1}\right)^{\lambda_1} \left[\left(1 - \frac{\mu_{\theta}'^2}{B_{\theta}/B_y}\right) (1 + \lambda_1 - n^2) + \frac{R_2}{R_1} \cot \alpha \left(\frac{B_{\theta}}{B_y} \frac{Q_{11,2}}{Z} \psi_1 + n \frac{Q_{12,2}}{Z} \Lambda_1 \right) \right] \\
 l_{22} &= \left(\frac{R_2}{R_1}\right)^{-\lambda_1} \left[\left(1 - \frac{\mu_{\theta}'^2}{B_{\theta}/B_y}\right) (1 - \lambda_1 - n^2) + \frac{R_2}{R_1} \cot \alpha \left(\frac{B_{\theta}}{B_y} \frac{Q_{11,2}}{Z} \psi_2 + n \frac{Q_{12,2}}{Z} \Lambda_2 \right) \right] \\
 l_{23} &= \left(\frac{R_2}{R_1}\right)^{\lambda_2} \left[\left(1 - \frac{\mu_{\theta}'^2}{B_{\theta}/B_y}\right) (1 + \lambda_2 - n^2) + \frac{R_2}{R_1} \cot \alpha \left(\frac{B_{\theta}}{B_y} \frac{Q_{11,2}}{Z} \psi_3 + n \frac{Q_{12,2}}{Z} \Lambda_3 \right) \right] \\
 l_{24} &= \left(\frac{R_2}{R_1}\right)^{-\lambda_2} \left[\left(1 - \frac{\mu_{\theta}'^2}{B_{\theta}/B_y}\right) (1 - \lambda_2 - n^2) + \frac{R_2}{R_1} \cot \alpha \left(\frac{B_{\theta}}{B_y} \frac{Q_{11,2}}{Z} \psi_4 + n \frac{Q_{12,2}}{Z} \Lambda_4 \right) \right] \\
 l_{31} &= -n \left(1 - \frac{\mu_{\theta}'^2}{B_{\theta}/B_y}\right) \lambda_1 - \cot \alpha \left(\frac{B_{\theta}}{B_y} \frac{Q_{21,1}}{Z} \psi_1 + n \frac{Q_{22,1}}{Z} \Lambda_1 \right) \\
 l_{32} &= -n \left(1 - \frac{\mu_{\theta}'^2}{B_{\theta}/B_y}\right) \lambda_1 - \cot \alpha \left(\frac{B_{\theta}}{B_y} \frac{Q_{21,1}}{Z} \psi_2 + n \frac{Q_{22,1}}{Z} \Lambda_2 \right) \\
 l_{33} &= -n \left(1 - \frac{\mu_{\theta}'^2}{B_{\theta}/B_y}\right) \lambda_2 - \cot \alpha \left(\frac{B_{\theta}}{B_y} \frac{Q_{21,1}}{Z} \psi_3 + n \frac{Q_{22,1}}{Z} \Lambda_3 \right)
 \end{aligned}$$

$$l_{34} = -n \left(1 - \frac{\mu_{\theta}'^2}{B_{\theta}/B_y} \right) \lambda_2 - \cot \alpha \left(\frac{B_{\theta}}{B_y} \frac{Q_{21,1}}{Z} \psi_4 + n \frac{Q_{22,1}}{Z} \Lambda_4 \right)$$

$$l_{41} = \left(\frac{R_2}{R_1} \right)^{\lambda_1} \left[n \left(1 - \frac{\mu_{\theta}'^2}{B_{\theta}/B_y} \right) \lambda_1 + \frac{R_2}{R_1} \cot \alpha \left(\frac{B_{\theta}}{B_y} \frac{Q_{21,2}}{Z} \psi_1 + n \frac{Q_{22,2}}{Z} \Lambda_1 \right) \right]$$

$$l_{42} = \left(\frac{R_2}{R_1} \right)^{-\lambda_1} \left[-n \left(1 - \frac{\mu_{\theta}'^2}{B_{\theta}/B_y} \right) \lambda_1 + \frac{R_2}{R_1} \cot \alpha \left(\frac{B_{\theta}}{B_y} \frac{Q_{21,2}}{Z} \psi_2 + n \frac{Q_{22,2}}{Z} \Lambda_2 \right) \right]$$

$$l_{43} = \left(\frac{R_2}{R_1} \right)^{\lambda_2} \left[n \left(1 - \frac{\mu_{\theta}'^2}{B_{\theta}/B_y} \right) \lambda_2 + \frac{R_2}{R_1} \cot \alpha \left(\frac{B_{\theta}}{B_y} \frac{Q_{21,2}}{Z} \psi_3 + n \frac{Q_{22,2}}{Z} \Lambda_3 \right) \right]$$

$$l_{44} = \left(\frac{R_2}{R_1} \right)^{-\lambda_2} \left[-n \left(1 - \frac{\mu_{\theta}'^2}{B_{\theta}/B_y} \right) \lambda_2 + \frac{R_2}{R_1} \cot \alpha \left(\frac{B_{\theta}}{B_y} \frac{Q_{12,1}}{Z} \psi_4 + n \frac{Q_{22,2}}{Z} \Lambda_4 \right) \right]$$

The expressions for g_{ij} are

$$g_{11} = \frac{\cot \alpha}{n^2 - 1} \left(n \frac{Q_{12,1}}{Z} - \frac{Q_{11,1}}{Z} \right) + \frac{Q_{13,1}}{Z}$$

$$g_{12} = \frac{n^2 - 1}{P_2} \left(1 - \frac{\mu_{\theta}'^2}{B_{\theta}/B_y} \right) + \cot \alpha \frac{Q_{11,1}}{Z} \left[\frac{n^2 (B_{\theta}/B_y) - \mu_{\theta}'}{P_2} \right] + n \cot \alpha \frac{Q_{12,1}}{Z} \left[\frac{1}{n^2 - 1} - \frac{n^2 (B_{\theta}/B_y) - \mu_{\theta}'}{P_2} \right] + \frac{Q_{14,1}}{Z} \tan \alpha$$

$$g_{1i} = \frac{-m\pi/x_1}{(m\pi/x_1)^4 - P_1(m\pi/x_1)^2 + P_2} \left(\left(1 - \frac{\mu_{\theta}'^2}{B_{\theta}/B_y} \right) \left[\left(\frac{m\pi}{x_1} \right)^2 + 1 - n^2 \right] + \cot \alpha \frac{Q_{11,1}}{Z} \left\{ \mu_{\theta}' \left[\left(\frac{m\pi}{x_1} \right)^2 + 1 \right] - \frac{B_{\theta}}{B_y} n^2 \right\} + n \cot \alpha \frac{Q_{12,1}}{Z} \left(\frac{B_{\theta}}{B_y} - \mu_{\theta}' \right) + \frac{Q_{14,1}}{Z} \frac{m\pi}{x_1} \tan \alpha \right) \quad (i > 2; \quad m = i - 2)$$

$$g_{21} = \frac{R_2}{R_1} \left[\cot \alpha \left(\frac{Q_{11,2}}{Z} - n \frac{Q_{12,2}}{Z} \right) - \frac{Q_{13,2}}{Z} \right]$$

$$g_{22} = \frac{n^2 - 1}{P_2} \left(1 - \frac{\mu'_\theta{}^2}{B_\theta/B_y} \right) + \frac{R_2}{R_1} \frac{\cot \alpha}{(n^2 - 1)^2} \frac{Q_{11,2}}{Z} \left[(n^2 - 1) \ln \frac{R_2}{R_1} - n^2 + \frac{\mu'_\theta}{B_\theta/B_y} \right] \\ - \frac{R_2}{R_1} n \cot \alpha \frac{Q_{12,2}}{Z} \left[\frac{1 + \ln R_2/R_1}{n^2 - 1} + \frac{n^2 (B_\theta/B_y) - \mu'_\theta}{P_2} \right] - \frac{R_2}{R_1} \frac{Q_{13,2}}{Z} \ln \frac{R_2}{R_1} - \frac{Q_{14,2}}{Z} \tan \alpha$$

$$g_{2i} = \frac{(-1)^m m\pi/x_1}{(m\pi/x_1)^4 - P_1(m\pi/x_1)^2 + P_2} \left(- \left(1 - \frac{\mu'_\theta{}^2}{B_\theta/B_y} \right) \left[\left(\frac{m\pi}{x_1} \right)^2 + 1 - n^2 \right] + \frac{R_2}{R_1} \cot \alpha \frac{Q_{11,2}}{Z} \right. \\ \left. \times \left\{ \mu'_\theta \left[\left(\frac{m\pi}{x_1} \right)^2 + 1 \right] - \frac{B_\theta}{B_y} n^2 \right\} + \frac{R_2}{R_1} n \cot \alpha \frac{Q_{12,2}}{Z} \left(\frac{B_\theta}{B_y} - \mu'_\theta \right) \right) \\ - \frac{Q_{14,2}}{Z} \tan \alpha (-1)^m \frac{m\pi}{x_1} \quad (i > 2; \quad m = i - 2)$$

$$g_{31} = \frac{\cot \alpha}{n^2 - 1} \left(n \frac{Q_{22,1}}{Z} - \frac{Q_{21,1}}{Z} \right) + \frac{Q_{23,1}}{Z}$$

$$g_{32} = \frac{\cot \alpha}{(n^2 - 1)^2} \frac{Q_{21,1}}{Z} \left(n^2 - \frac{\mu'_\theta}{B_\theta/B_y} \right) + n \cot \alpha \frac{Q_{22,1}}{Z} \left[\frac{1}{n^2 - 1} - \frac{\left(n^2 - \frac{\mu'_\theta}{B_\theta/B_y} \right)}{(n^2 - 1)^2} \right] + \frac{Q_{24,1}}{Z} \tan \alpha$$

$$g_{3i} = \frac{-m\pi/x_1}{(m\pi/x_1)^4 - P_1(m\pi/x_1)^2 + P_2} \left(-n \left(1 - \frac{\mu'_\theta{}^2}{B_\theta/B_y} \right) \left(\frac{m\pi}{x_1} \right)^2 + \cot \alpha \frac{Q_{21,1}}{Z} \left\{ \mu'_\theta \left[\left(\frac{m\pi}{x_1} \right)^2 + 1 \right] \right. \right. \\ \left. \left. - \frac{B_\theta}{B_y} n^2 \right\} + n \cot \alpha \frac{Q_{22,1}}{Z} \left(\frac{B_\theta}{B_y} - \mu'_\theta \right) \right) + \frac{Q_{24,1}}{Z} \tan \alpha \frac{m\pi}{x_1} \quad (i > 2; \quad m = i - 2)$$

$$g_{41} = \frac{R_2}{R_1} \left[\frac{\cot \alpha}{n^2 - 1} \left(\frac{Q_{21,2}}{Z} - n \frac{Q_{22,2}}{Z} \right) - \frac{Q_{23,2}}{Z} \right]$$

$$g_{42} = \frac{R_2}{R_1} \frac{\cot \alpha}{(n^2 - 1)^2} \frac{Q_{21,2}}{Z} \left[(n^2 - 1) \ln \frac{R_2}{R_1} - \left(n^2 - \frac{\mu'_\theta}{B_\theta/B_y} \right) \right] - \frac{R_2}{R_1} n \cot \alpha \frac{Q_{22,2}}{Z} \left[\frac{1 + \ln R_2/R_1}{n^2 - 1} \right. \\ \left. - \frac{n^2(B_\theta/B_y) - \mu'_\theta}{P_2} \right] - \frac{R_2}{R_1} \frac{Q_{23,2}}{Z} \ln \frac{R_2}{R_1} - \frac{Q_{24,2}}{Z} \tan \alpha$$

$$g_{4i} = \frac{(-1)^m m\pi/x_1}{(m\pi/x_1)^4 - P_1(m\pi/x_1)^2 + P_2} \left(n \left(1 - \frac{\mu'^2_\theta}{B_\theta/B_y} \right) \left(\frac{m\pi}{x_1} \right)^2 + \frac{R_2}{R_1} \cot \alpha \frac{Q_{21,2}}{Z} \left\{ \mu'_\theta \left[\left(\frac{m\pi}{x_1} \right)^2 + 1 \right] \right. \right. \right. \\ \left. \left. - \frac{B_\theta}{B_y} n^2 \right\} + \frac{R_2}{R_1} n \cot \alpha \frac{Q_{22,2}}{Z} \left(\frac{B_\theta}{B_y} - \mu'_\theta \right) \right) - \frac{Q_{24,2}}{Z} \tan \alpha (-1)^m \frac{m\pi}{x_1} \\ (i > 2; \quad m = i - 2)$$

The expressions for h_{ij} are

$$h_{11} = 1 - \frac{\mu'^2_\theta}{B_\theta/B_y} - \frac{Q_{11,1}}{Z} \cot \alpha \left(\sqrt{B_\theta/B_y} - \mu'_\theta \right)$$

$$h_{12} = 1 - \frac{\mu'^2_\theta}{B_\theta/B_y} + \frac{Q_{11,1}}{Z} \cot \alpha \left(\sqrt{B_\theta/B_y} + \mu'_\theta \right)$$

$$h_{21} = \left(\frac{R_2}{R_1} \right)^{\lambda_0} \left[1 - \frac{\mu'^2_\theta}{B_\theta/B_y} + \frac{R_2}{R_1} \frac{Q_{11,2}}{Z} \cot \alpha \left(\sqrt{B_\theta/B_y} - \mu'_\theta \right) \right]$$

$$h_{22} = \left(\frac{R_2}{R_1} \right)^{-\lambda_0} \left[1 - \frac{\mu'^2_\theta}{B_\theta/B_y} - \frac{R_2}{R_1} \frac{Q_{11,2}}{Z} \cot \alpha \left(\sqrt{B_\theta/B_y} + \mu'_\theta \right) \right]$$

The expressions for \bar{e}_{ij} are

$$\bar{e}_{11} = \frac{Q_{11,1}}{Z} \cot \alpha + \frac{Q_{13,1}}{Z}$$

$$\bar{e}_{12} = \frac{1 - \mu'_\theta \mu'_y}{B_\theta/B_y} - \frac{Q_{11,1}}{Z} \cot \alpha \frac{\mu'_\theta}{B_\theta/B_y} + \frac{Q_{14,1}}{Z} \tan \alpha$$

$$\bar{e}_{1j} = \frac{-m\pi/x_1}{\left(\frac{m\pi}{x_1}\right)^2 + (B_\theta/B_y)} \left(1 - \mu'_\theta \mu'_y + \frac{Q_{11,1}}{Z} \cot \alpha \mu'_\theta\right) + \frac{m\pi}{x_1} \frac{Q_{14,1}}{Z} \tan \alpha \quad (j > 2; \quad m = j - 2)$$

$$\bar{e}_{21} = -\frac{R_2}{R_1} \left(\frac{Q_{11,2}}{Z} \cot \alpha + \frac{Q_{13,2}}{Z} \right)$$

$$\bar{e}_{22} = - \left[\frac{1 - \mu'_\theta \mu'_y}{B_\theta/B_y} + \frac{R_2}{R_1} \cot \alpha \frac{Q_{11,2}}{Z} \left(\ln \frac{R_2}{R_1} - \frac{\mu'_\theta}{B_\theta/B_y} \right) + \frac{R_2}{R_1} \ln \frac{R_2}{R_1} \frac{Q_{13,2}}{Z} + \frac{Q_{14,2}}{Z} \tan \alpha \right]$$

$$\bar{e}_{2j} = (-1)^m \frac{m\pi}{x_1} \frac{(R_2/R_1) \cot \alpha \mu'_\theta Q_{11,2} - (1 - \mu'_\theta \mu'_y)}{\left(m\pi/x_1\right)^2 + (B_\theta/B_y)} - \frac{Q_{14,2}}{Z} \tan \alpha \quad (j > 2; \quad m = j - 2)$$

where

$$Z = \left(\frac{R_1}{\cos \alpha} \right)^2 \frac{B_\theta}{D_y}$$

$$Q_{ij} = S_{ij} - \frac{C_\omega M_{ij}}{\cos^4 \alpha}$$

EXPRESSIONS FOR b_{ij} , c_{ij} , d_{ij} , e_{ij} , AND f_{ij}

The expressions for b_{ij} are

$$b_{11} = n^2 \left[1 - \left(\frac{R_2}{R_1} \right)^{-2} \right] \left(\frac{n^2}{2} \frac{D_\theta}{D_y} + \frac{D_{y\theta}}{D_y} \right) - Z(1 - \mu'_\theta \mu'_y) t_1 \cot^4 \alpha + \gamma_1$$

$$b_{12} = \frac{n^2}{4} \left[1 - \left(\frac{R_2}{R_1} \right)^{-2} \right] \left[\frac{D_\theta}{D_y} (n^2 - 2) - 2 \left(\frac{D_{y\theta}}{D_y} - \mu_\theta \right) \right] - n^2 \left(\frac{n^2}{2} \frac{D_\theta}{D_y} + \frac{D_{y\theta}}{D_y} \right) \left(\frac{R_2}{R_1} \right)^{-2} \ln \frac{R_2}{R_1} \\ - Z(1 - \mu'_\theta \mu'_y) t_2 \cot^4 \alpha + \gamma_2$$

$$b_{21} = b_{12}$$

with t_2 and γ_2 replaced by t_4 and γ_4 .

$$b_{1j} = n^2 \frac{m\pi}{\ln R_2/R_1} \frac{1 - (-1)^m (R_2/R_1)^{-2}}{4 + \left(\frac{m\pi}{\ln R_2/R_1} \right)^2} \left\{ \mu_\theta \left[\left(\frac{m\pi}{\ln R_2/R_1} \right)^2 + 2 \right] + \frac{D_\theta}{D_y} (n^2 - 2) - 2 \frac{D_{y\theta}}{D_y} \right\} \\ - Z(1 - \mu'_\theta \mu'_y) t_{3m} \cot^4 \alpha + \gamma_{3m} \quad (j > 2; \quad m = j - 2)$$

$$b_{22} = \frac{1}{4} \left[1 - \left(\frac{R_2}{R_1} \right)^{-2} \right] \left[\frac{D_\theta}{D_y} (n^4 - 2n^2 + 2) + 2n^2 \frac{D_{y\theta}}{D_y} + 2\mu_\theta (n^2 - 2) + 2 \right] - n^2 \left(\frac{R_2}{R_1} \right)^{-2} \ln \frac{R_2}{R_1} \\ \times \left[\frac{D_\theta}{D_y} \left(\frac{n^2}{2} - 1 \right) + \mu_\theta - \frac{D_{y\theta}}{D_y} \right] - \frac{n^2}{2} \left(\frac{R_2}{R_1} \right)^{-2} \left(\ln \frac{R_2}{R_1} \right)^2 \left(n^2 \frac{D_\theta}{D_y} + 2 \frac{D_{y\theta}}{D_y} \right) \\ - Z(1 - \mu'_\theta \mu'_y) t_5 \cot^4 \alpha + \gamma_5$$

$$b_{2j} = \frac{m\pi}{\ln R_2/R_1} \left[\frac{n^2 (-1)^m (R_2/R_1)^{-2} \ln R_2/R_1}{4 + \left(\frac{m\pi}{\ln R_2/R_1} \right)^2} \left\{ \frac{D_\theta}{D_y} (2 - n^2) + 2 \frac{D_{y\theta}}{D_y} - \mu_\theta \left[2 + \left(\frac{m\pi}{\ln R_2/R_1} \right)^2 \right] \right\} \right]$$

(Equation continued on next page)

$$\begin{aligned}
& + \frac{1 - (-1)^m (R_2/R_1)^{-2}}{\left[4 + \left(\frac{m\pi}{\ln R_2/R_1}\right)^2\right]^2} \left(\mu_\theta \left\{ n^2 \left[8 + 4 \left(\frac{m\pi}{\ln R_2/R_1} \right)^2 \right] - \left[4 + \left(\frac{m\pi}{\ln R_2/R_1} \right)^2 \right]^2 \right\} + \frac{D_\theta}{D_y} \left[2 \left(\frac{m\pi}{\ln R_2/R_1} \right)^2 \right. \right. \\
& \left. \left. + 8(1 - n^2) + 4n^4 \right] + n^2 \frac{D_{y\theta}}{D_y} \left[8 + 4 \left(\frac{m\pi}{\ln R_2/R_1} \right)^2 \right] - 2 \left(\frac{m\pi}{\ln R_2/R_1} \right)^2 - 8 + \left[4 + \left(\frac{m\pi}{\ln R_2/R_1} \right)^2 \right]^2 \right) \\
& - Z \left(1 - \mu'_\theta \mu'_y \right) t_{6m} \cot^4 \alpha + \gamma_{6m} \quad (j > 2; \quad m = j - 2)
\end{aligned}$$

$$b_{i1} = b_{1j} \quad (i > 2; \quad p = i - 2)$$

with t_{3m} and γ_{3m} replaced by t_{7p} and γ_{7p} and m replaced by p .

$$b_{i2} = b_{2j} \quad (i > 2; \quad p = i - 2)$$

with t_{6m} and γ_{6m} replaced by t_{8p} and γ_{8p} and m replaced by p .

$$\begin{aligned}
b_{jj} = & \left(\frac{p\pi}{\ln R_2/R_1} \right)^2 \left(\frac{1 - (R_2/R_1)^{-2}}{4 \left[1 + \left(\frac{p\pi}{\ln R_2/R_1} \right)^2 \right]} \left\{ \frac{D_\theta}{D_y} \left[\left(\frac{p\pi}{\ln R_2/R_1} \right)^2 + 2 + n^2(n^2 - 2) \right] \right. \right. \\
& \left. \left. + 2n^2 \frac{D_{y\theta}}{D_y} \left[1 + \left(\frac{p\pi}{\ln R_2/R_1} \right)^2 \right] + 2n^2 \mu_\theta \left[1 + \left(\frac{p\pi}{\ln R_2/R_1} \right)^2 \right] + \left(\frac{p\pi}{\ln R_2/R_1} \right)^4 - \left(\frac{p\pi}{\ln R_2/R_1} \right)^2 - 2 \right\} \right. \\
& \left. + \left[1 - \left(\frac{R_2}{R_1} \right)^{-2} \right] (1 - \mu_\theta) \right) - Z \left(1 - \mu'_\theta \mu'_y \right) t_{9p} \cot^4 \alpha + \gamma_{9p} \quad (j > 2; \quad p = j - 2)
\end{aligned}$$

$$\begin{aligned}
b_{ij} = & \frac{m\pi}{\ln R_2/R_1} \frac{p\pi}{\ln R_2/R_1} \left[\frac{1 - (-1)^{m+p} \left(\frac{R_2}{R_1}\right)^{-2}}{\left\{ 4 + \left[\frac{(m-p)\pi}{\ln R_2/R_1} \right]^2 \right\} \left\{ 4 + \left[\frac{(m+p)\pi}{\ln R_2/R_1} \right]^2 \right\}} \right. \\
& \times \left\{ \left(\frac{D_\theta}{D_y} - 1 \right) \left[8 + 2 \left(\frac{m\pi}{\ln R_2/R_1} \right)^2 + 2 \left(\frac{p\pi}{\ln R_2/R_1} \right)^2 \right] + 4n^2 \frac{D_\theta}{D_y} (n^2 - 2) + 4n^2 \left(\frac{D_y \theta}{D_y} + \mu_\theta \right) \right. \\
& \times \left. \left[2 + \left(\frac{m\pi}{\ln R_2/R_1} \right)^2 + \left(\frac{p\pi}{\ln R_2/R_1} \right)^2 \right] + 4 \left(\frac{m\pi}{\ln R_2/R_1} \right)^2 \left(\frac{p\pi}{\ln R_2/R_1} \right)^2 + 1 - \mu_\theta \right] \\
& \left. - Z \left(1 - \mu'_\theta \mu'_y \right) t_{9mp} \cot^4 \alpha + \gamma_{9mp} \right. \\
& \left. (i, j > 2; \quad m = j - 2; \quad p = i - 2) \right.
\end{aligned}$$

The expressions for c_{ij} are

$$c_{11} = c_{12} = c_{21} = c_{1j} = c_{i1} = 0$$

$$c_{22} = - \left[1 - \left(\frac{R_2}{R_1} \right)^{-1} \right]$$

$$c_{2j} = - \frac{m\pi}{\ln R_2/R_1} \left[\frac{1 - (-1)^m \left(\frac{R_2}{R_1} \right)^{-1}}{1 + \left(\frac{m\pi}{\ln R_2/R_1} \right)^2} \right] = c_{i2} \quad (j > 2; \quad m = j - 2)$$

$$c_{jj} = - \frac{\left(\frac{p\pi}{\ln R_2/R_1} \right)^2 \left[1 + 2 \left(\frac{p\pi}{\ln R_2/R_1} \right)^2 \right] \left[1 - \left(\frac{R_2}{R_1} \right)^{-1} \right]}{1 + 4 \left(\frac{p\pi}{\ln R_2/R_1} \right)^2} \quad (j > 2; \quad p = j - 2)$$

$$c_{ij} = -\frac{m\pi}{\ln R_2/R_1} \frac{p\pi}{\ln R_2/R_1} \frac{1 - (-1)^{m+p} (R_2/R_1)^{-1} \left[1 + \left(\frac{m\pi}{\ln R_2/R_1} \right)^2 + \left(\frac{p\pi}{\ln R_2/R_1} \right)^2 \right]}{\left\{ 1 + \left[\frac{(m-p)\pi}{\ln R_2/R_1} \right]^2 \right\} \left\{ 1 + \left[\frac{(m+p)\pi}{\ln R_2/R_1} \right]^2 \right\}}$$

(i, j > 2; m = j - 2; p = i - 2)

The expressions for d_{ij} are

$$d_{11} = -n^2 \left(\frac{R_2}{R_1} - 1 \right)$$

$$d_{12} = d_{21} = -n^2 \left[\frac{R_2}{R_1} \left(\ln \frac{R_2}{R_1} - 1 \right) + 1 \right]$$

$$d_{1j} = -n^2 \frac{m\pi}{\ln R_2/R_1} \left[\frac{1 - (-1)^m (R_2/R_1)}{1 + \left(\frac{m\pi}{\ln R_2/R_1} \right)^2} \right] = d_{i1} \quad (j > 2; m = j - 2)$$

with m replaced by p .

$$d_{22} = \frac{1}{2} \left(1 - \frac{R_2}{R_1} \right) - n^2 \left[2 \left(\frac{R_2}{R_1} - 1 \right) + \frac{R_2}{R_1} \ln \frac{R_2}{R_1} \left(\ln \frac{R_2}{R_1} - 2 \right) \right]$$

$$d_{2j} = \frac{m\pi}{\ln R_2/R_1} \left\{ \frac{n^2 (-1)^m (R_2/R_1) \ln R_2/R_1 + \frac{1}{2} [1 - (-1)^m (R_2/R_1)]}{1 + \left(\frac{m\pi}{\ln R_2/R_1} \right)^2} \right. \\ \left. + \frac{2n^2 [1 - (-1)^m (R_2/R_1)]}{\left[1 + \left(\frac{m\pi}{\ln R_2/R_1} \right)^2 \right]^2} \right\} = d_{i2} \quad (j > 2; m = j - 2)$$

with m replaced by p .

$$d_{jj} = \frac{1}{2} \left(\frac{p\pi}{\ln R_2/R_1} \right)^2 \left\{ \frac{\left[1 + 2 \left(\frac{p\pi}{\ln R_2/R_1} \right)^2 \right] \left[1 - (R_2/R_1) \right] + 4n^2 \left[1 - (R_2/R_1) \right]}{1 + 4 \left(\frac{p\pi}{\ln R_2/R_1} \right)^2} \right\}$$

(j > 2; p = j - 2)

$$d_{ij} = \frac{1}{2} \frac{m\pi}{\ln R_2/R_1} \frac{p\pi}{\ln R_2/R_1} \frac{\left[1 + \left(\frac{m\pi}{\ln R_2/R_1} \right)^2 + \left(\frac{p\pi}{\ln R_2/R_1} \right)^2 + 4n^2 \right] \left[1 - (-1)^{m+p} (R_2/R_1) \right]}{\left\{ 1 + \left[\frac{(m-p)\pi}{\ln R_2/R_1} \right]^2 \right\} \left\{ 1 + \left[\frac{(m+p)\pi}{\ln R_2/R_1} \right]^2 \right\}}$$

(i, j > 2; m = i - 2; p = j - 2)

The expressions for e_{ij} are

$$e_{11} = \frac{1}{2} \left[\left(\frac{R_2}{R_1} \right)^2 - 1 \right] + \beta_1$$

$$e_{12} = \frac{1}{4} \left[\left(\frac{R_2}{R_1} \right)^2 \left(2 \ln \frac{R_2}{R_1} - 1 \right) + 1 \right] + \beta_2$$

$$e_{1j} = \frac{m\pi}{\ln R_2/R_1} \frac{\left[1 - (-1)^m (R_2/R_1)^2 \right]}{4 + \left(\frac{m\pi}{\ln R_2/R_1} \right)^2} + \beta_{3m}$$

(j > 2; m = j - 2)

$$e_{21} = e_{12}$$

with β_2 replaced by β_4 .

$$e_{22} = \frac{1}{4} \left\{ \left(\frac{R_2}{R_1} \right)^2 \left[2 \left(\ln \frac{R_2}{R_1} \right)^2 - 2 \ln \frac{R_2}{R_1} + 1 \right] + 1 \right\} + \beta_5$$

$$e_{2j} = \frac{m\pi}{\ln R_2/R_1} \left\{ - \frac{(-1)^m (R_2/R_1)^2 \ln R_2/R_1}{4 + \left(\frac{m\pi}{\ln R_2/R_1} \right)^2} + \frac{4 \left[(-1)^m (R_2/R_1)^2 - 1 \right]}{\left[4 + \left(\frac{m\pi}{\ln R_2/R_1} \right)^2 \right]^2} \right\} + \beta_{6m}$$

(j > 2; m = j - 2)

$$e_{i1} = e_{1j} \quad (i > 2; \quad p = i - 2)$$

with β_{3m} replaced by β_{7p} and m replaced by p .

$$e_{i2} = e_{2j} \quad (i > 2; \quad p = i - 2)$$

with β_{6m} replaced by β_{8m} and m replaced by p .

$$e_{ij} = 4 \frac{p\pi}{\ln R_2/R_1} \frac{m\pi}{\ln R_2/R_1} \frac{(-1)^{p+m} (R_2/R_1)^2 - 1}{\left\{ 4 + \left[(p+m) \frac{\pi}{\ln R_2/R_1} \right]^2 \right\} \left\{ 4 + \left[(p-m) \frac{\pi}{\ln R_2/R_1} \right]^2 \right\}} + \beta_{9mp}$$

(i, j > 2; $m = i - 2$; $p = j - 2$)

The expressions for f_{ij} are

$$f_{11} = f_{21} = f_{i1} = 0$$

$$f_{12} = \frac{R_2}{R_1} - 1$$

$$f_{1j} = - \frac{m\pi}{\ln R_2/R_1} \frac{1 - (-1)^m (R_2/R_1)}{1 + \left(\frac{m\pi}{\ln R_2/R_1} \right)^2} \quad (j > 2; \quad m = j - 2)$$

$$f_{22} = \frac{R_2}{R_1} \left(\ln \frac{R_2}{R_1} - 1 \right) + 1$$

$$f_{2j} = \frac{m\pi}{x_1} \left\{ \frac{(-1)^m (R_2/R_1) \ln (R_2/R_1)}{1 + \left(\frac{m\pi}{\ln R_2/R_1} \right)^2} + \frac{\left[1 - \left(\frac{m\pi}{\ln R_2/R_1} \right)^2 \right] \left[1 - (-1)^m (R_2/R_1) \right]}{\left[1 + \left(\frac{m\pi}{\ln R_2/R_1} \right)^2 \right]^2} \right\}$$

(j > 2; $m = j - 2$)

$$f_{i2} = \frac{m\pi}{\ln R_2/R_1} \frac{1 - (-1)^m (R_2/R_1)}{1 + \left(\frac{m\pi}{\ln R_2/R_1} \right)^2} \quad (i > 2; \quad m = i - 2)$$

$$f_{ii} = - \frac{\left(\frac{m\pi}{\ln R_2/R_1}\right)^2 \left(\frac{R_2}{R_1} - 1\right)}{1 + 4\left(\frac{m\pi}{\ln R_2/R_1}\right)^2} \quad (i > 2; \quad m = i - 2)$$

$$f_{ij} = \frac{\frac{m\pi}{\ln R_2/R_1} \frac{p\pi}{\ln R_2/R_1} \left[1 - (-1)^{p+m} \left(\frac{R_2}{R_1}\right)\right] \left[1 + \left(\frac{p\pi}{\ln R_2/R_1}\right)^2 - \left(\frac{m\pi}{\ln R_2/R_1}\right)^2\right]}{\left\{1 + \left[\frac{(p+m)\pi}{\ln R_2/R_1}\right]^2\right\} \left\{1 + \left[\frac{(p-m)\pi}{\ln R_2/R_1}\right]^2\right\}} \quad (i, j > 2; \quad m = i - 2; \quad p = j - 2)$$

The quantities t_i are defined as

$$t_1 = (\lambda_1 + 1) \Omega_1 \left[\left(\frac{R_2}{R_1}\right)^{\lambda_1} - 1 \right] + (\lambda_1 - 1) \Omega_2 \left[1 - \left(\frac{R_2}{R_1}\right)^{-\lambda_1} \right] + (\lambda_2 + 1) \Omega_3 \left[\left(\frac{R_2}{R_1}\right)^{\lambda_2} - 1 \right] \\ + (\lambda_2 - 1) \Omega_4 \left[1 - \left(\frac{R_2}{R_1}\right)^{-\lambda_2} \right]$$

$$t_2 = t_1$$

with Ω_i replaced by Γ_i .

$$t_3 = t_1$$

with Ω_i replaced by Φ_{im} and the additional term

$$- \frac{\frac{m\pi}{\ln R_2/R_1} \left[\left(\frac{m\pi}{\ln R_2/R_1}\right)^2 + 1 \right] \left[1 - (-1)^m \right]}{\left(\frac{m\pi}{\ln R_2/R_1}\right)^4 - P_1 \left(\frac{m\pi}{\ln R_2/R_1}\right)^2 + P_2}$$

$$t_4 = \frac{\lambda_1 + 1}{\lambda_1} \Omega_1 \left[1 + \left(\frac{R_2}{R_1}\right)^{\lambda_1} \left(\lambda_1 \ln \frac{R_2}{R_1} - 1 \right) \right] + \frac{\lambda_1 - 1}{\lambda_1} \Omega_2 \left[1 - \left(\frac{R_2}{R_1}\right)^{-\lambda_1} \left(\lambda_1 \ln \frac{R_2}{R_1} + 1 \right) \right]$$

(Equation continued on next page)

$$+ \frac{\lambda_2 + 1}{\lambda_2} \Omega_3 \left[1 + \left(\frac{R_2}{R_1} \right)^{\lambda_2} \left(\lambda_2 \ln \frac{R_2}{R_1} - 1 \right) \right] + \frac{\lambda_2 - 1}{\lambda_2} \Omega_4 \left[1 - \left(\frac{R_2}{R_1} \right)^{-\lambda_2} \left(\lambda_2 \ln \frac{R_2}{R_1} + 1 \right) \right]$$

$$t_5 = t_4$$

with Ω_i replaced by Γ_i .

$$t_{6m} = t_4$$

with Ω_i replaced by Φ_{im} and the additional term

$$+ \frac{\frac{m\pi}{\ln R_2/R_1} \left[\left(\frac{m\pi}{\ln R_2/R_1} \right)^2 + 1 \right] (-1)^m \ln R_2/R_1}{\left(\frac{m\pi}{\ln R_2/R_1} \right)^4 - P_1 \left(\frac{m\pi}{\ln R_2/R_1} \right)^2 + P_2}$$

$$t_{7p} = \frac{p\pi}{\ln R_2/R_1} \left\{ \frac{\lambda_1(\lambda_1 + 1) \Omega_1 \left[1 - (-1)^p \left(R_2/R_1 \right)^{\lambda_1} \right]}{\lambda_1^2 + \left(\frac{p\pi}{\ln R_2/R_1} \right)^2} + \frac{\lambda_1(\lambda_1 - 1) \Omega_2 \left[1 - (-1)^p \left(R_2/R_1 \right)^{-\lambda_1} \right]}{\lambda_1^2 + \left(\frac{p\pi}{\ln R_2/R_1} \right)^2} \right. \\ \left. + \frac{\lambda_2(\lambda_2 + 1) \Omega_3 \left[1 - (-1)^p \left(R_2/R_1 \right)^{\lambda_2} \right]}{\lambda_2^2 + \left(\frac{p\pi}{\ln R_2/R_1} \right)^2} + \frac{\lambda_2(\lambda_2 - 1) \Omega_4 \left[1 - (-1)^p \left(R_2/R_1 \right)^{-\lambda_2} \right]}{\lambda_2^2 + \left(\frac{p\pi}{\ln R_2/R_1} \right)^2} \right\}$$

$$t_{8p} = t_{7p}$$

with Ω_i replaced by Γ_i .

$$t_{9mp} = t_{7p}$$

with Ω_i replaced by Φ_{im} .

$$t_{9p} = t_{7p}$$

with Ω_i replaced by Φ_{ip} and the additional term

$$- \frac{\frac{1}{2} \left(\frac{p\pi}{\ln R_2/R_1} \right)^2 \left[\left(\frac{p\pi}{\ln R_2/R_1} \right)^2 + 1 \right] \ln R_2/R_1}{\left(\frac{p\pi}{\ln R_2/R_1} \right)^4 - P_1 \left(\frac{p\pi}{\ln R_2/R_1} \right)^2 + P_2}$$

The quantities γ are defined as

$$\begin{aligned} \gamma_1 = & \cot^3 \alpha \left(S_{33,1} + \frac{R_2}{R_1} S_{33,2} \right) + \cot^4 \alpha \frac{B_\theta}{B_y} \left\{ S_{31,1} \left[\Omega_1 \psi_1 + \Omega_2 \psi_2 + \Omega_3 \psi_3 + \Omega_4 \psi_4 \right. \right. \\ & - \left. \frac{1}{(B_\theta/B_y)(n^2 - 1)} \right] + S_{31,2} \frac{R_2}{R_1} \left[\Omega_1 \psi_1 \left(\frac{R_2}{R_1} \right)^{\lambda_1} + \Omega_2 \psi_2 \left(\frac{R_2}{R_1} \right)^{-\lambda_1} + \Omega_3 \psi_3 \left(\frac{R_2}{R_1} \right)^{\lambda_2} \right. \\ & + \left. \Omega_4 \psi_4 \left(\frac{R_2}{R_1} \right)^{-\lambda_2} - \frac{1}{(B_\theta/B_y)(n^2 - 1)} \right] \left. \right\} + n \cot^4 \alpha \left\{ S_{32,1} \left(\Lambda_1 \Omega_1 + \Lambda_2 \Omega_2 + \Lambda_3 \Omega_3 \right. \right. \\ & + \left. \Lambda_4 \Omega_4 + \frac{1}{n^2 - 1} \right) + S_{32,2} \frac{R_2}{R_1} \left[\Lambda_1 \Omega_1 \left(\frac{R_2}{R_1} \right)^{\lambda_1} + \Lambda_2 \Omega_2 \left(\frac{R_2}{R_1} \right)^{-\lambda_1} + \Lambda_3 \Omega_3 \left(\frac{R_2}{R_1} \right)^{\lambda_2} \right. \\ & + \left. \Lambda_4 \Omega_4 \left(\frac{R_2}{R_1} \right)^{-\lambda_2} + \frac{1}{n^2 - 1} \right] \left. \right\} \\ \gamma_2 = & \frac{R_2}{R_1} \ln \frac{R_2}{R_1} \cot^3 \alpha S_{33,2} + \frac{B_\theta}{B_y} \cot^4 \alpha \left\{ S_{31,1} \left(\Gamma_1 \psi_1 + \Gamma_2 \psi_2 + \Gamma_3 \psi_3 + \Gamma_4 \psi_4 + \frac{n^2 - \mu'_y}{P_2} \right) \right. \\ & + S_{31,2} \frac{R_2}{R_1} \left[\Gamma_1 \psi_1 \left(\frac{R_2}{R_1} \right)^{\lambda_1} + \Gamma_2 \psi_2 \left(\frac{R_2}{R_1} \right)^{-\lambda_1} + \Gamma_3 \psi_3 \left(\frac{R_2}{R_1} \right)^{\lambda_2} + \Gamma_4 \psi_4 \left(\frac{R_2}{R_1} \right)^{-\lambda_2} \right. \\ & - \left. \frac{\ln R_2/R_1}{(B_\theta/B_y)(n^2 - 1)} + \frac{n^2 - \mu'_y}{P_2} \right] \left. \right\} + n \cot^4 \alpha \left\{ S_{32,1} \left(\Lambda_1 \Gamma_1 + \Lambda_2 \Gamma_2 + \Lambda_3 \Gamma_3 + \Lambda_4 \Gamma_4 \right. \right. \\ & \left. \left. \text{(Equation continued on next page)} \right) \right\} \end{aligned}$$

$$\begin{aligned}
& - \frac{1 - \frac{\mu'_\theta}{B_\theta/B_y}}{(n^2 - 1)^2} + S_{32,2} \frac{R_2}{R_1} \left[\Lambda_1 \Gamma_1 \left(\frac{R_2}{R_1} \right)^{\lambda_1} + \Lambda_2 \Gamma_2 \left(\frac{R_2}{R_1} \right)^{-\lambda_1} + \Lambda_3 \Gamma_3 \left(\frac{R_2}{R_1} \right)^{\lambda_2} + \Lambda_4 \Gamma_4 \left(\frac{R_2}{R_1} \right)^{-\lambda_2} \right. \\
& \left. + \frac{\ln R_2/R_1}{n^2 - 1} - \frac{1 - \frac{\mu'_\theta}{B_\theta/B_y}}{(n^2 - 1)^2} \right] + \cot^2 \alpha (S_{34,1} + S_{34,2}) \\
\gamma_{3m} = & \frac{B_\theta}{B_y} \cot^4 \alpha \left[S_{31,1} \left(\Phi_{1m} \psi_1 + \Phi_{2m} \psi_2 + \Phi_{3m} \psi_3 + \Phi_{4m} \psi_4 \right. \right. \\
& - \frac{\frac{m\pi}{\ln R_2/R_1} \left\{ \mu'_y \left[\left(\frac{m\pi}{\ln R_2/R_1} \right)^2 + 1 \right] - n^2 \right\}}{\left(\frac{m\pi}{\ln R_2/R_1} \right)^4 - P_1 \left(\frac{m\pi}{\ln R_2/R_1} \right)^2 + P_2} + S_{31,2} \frac{R_2}{R_1} \left(\Phi_{1m} \psi_1 \left(\frac{R_2}{R_1} \right)^{\lambda_1} + \Phi_{2m} \psi_2 \left(\frac{R_2}{R_1} \right)^{-\lambda_1} \right. \\
& \left. \left. + \Phi_{3m} \psi_3 \left(\frac{R_2}{R_1} \right)^{\lambda_2} + \Phi_{4m} \psi_4 \left(\frac{R_2}{R_1} \right)^{-\lambda_2} - \frac{(-1)^m \frac{m\pi}{\ln R_2/R_1} \left\{ \mu'_y \left[\left(\frac{m\pi}{\ln R_2/R_1} \right)^2 + 1 \right] - n^2 \right\}}{\left(\frac{m\pi}{\ln R_2/R_1} \right)^4 - P_1 \left(\frac{m\pi}{\ln R_2/R_1} \right)^2 + P_2} \right) \right] \\
& + n \cot^4 \alpha \left\{ S_{32,1} \left[\Lambda_1 \Phi_{1m} + \Lambda_2 \Phi_{2m} + \Lambda_3 \Phi_{3m} + \Lambda_4 \Phi_{4m} \right. \right.
\end{aligned}$$

(Equation continued on next page)

$$\begin{aligned}
& - \frac{\frac{m\pi}{\ln R_2/R_1} \left(\frac{B_\theta}{B_y} - \mu'_\theta \right)}{\left(\frac{m\pi}{\ln R_2/R_1} \right)^4 - P_1 \left(\frac{m\pi}{\ln R_2/R_1} \right)^2 + P_2} \left[+ S_{32,2} \frac{R_2}{R_1} \left[\Lambda_1 \Phi_{1m} \left(\frac{R_2}{R_1} \right)^{\lambda_1} + \Lambda_2 \Phi_{2m} \left(\frac{R_2}{R_1} \right)^{-\lambda_1} \right. \right. \\
& \left. \left. + \Lambda_3 \Phi_{3m} \left(\frac{R_2}{R_1} \right)^{\lambda_2} + \Lambda_4 \Phi_{4m} \left(\frac{R_2}{R_1} \right)^{-\lambda_2} - \frac{(-1)^m \frac{m\pi}{\ln R_2/R_1} \left(\frac{B_\theta}{B_y} - \mu'_\theta \right)}{\left(\frac{m\pi}{\ln R_2/R_1} \right)^4 - P_1 \left(\frac{m\pi}{\ln R_2/R_1} \right)^2 + P_2} \right] \right. \\
& \left. + \frac{m\pi}{\ln R_2/R_1} \cot^2 \alpha \left[S_{34,1} + (-1)^m S_{34,2} \right] \right\} \\
\gamma_4 = & \frac{R_2}{R_1} \ln \frac{R_2}{R_1} \cot^3 \alpha S_{33,2} + S_{31,2} \frac{R_2}{R_1} \ln \frac{R_2}{R_1} \cot^4 \alpha \frac{B_\theta}{B_y} \left[\Omega_1 \psi_1 \left(\frac{R_2}{R_1} \right)^{\lambda_1} + \Omega_2 \psi_2 \left(\frac{R_2}{R_1} \right)^{-\lambda_1} \right. \\
& \left. + \Omega_3 \psi_3 \left(\frac{R_2}{R_1} \right)^{\lambda_2} + \Omega_4 \psi_4 \left(\frac{R_2}{R_1} \right)^{-\lambda_2} - \frac{1}{(B_\theta/B_y)(n^2 - 1)} \right] + S_{41,1} \cot^3 \alpha \frac{B_\theta}{B_y} \left[\Omega_1 \psi_1 + \Omega_2 \psi_2 \right. \\
& \left. + \Omega_3 \psi_3 + \Omega_4 \psi_4 + \frac{1}{(B_\theta/B_y)(1 - n^2)} \right] + n S_{42,1} \cot^3 \alpha \left(\Lambda_1 \Omega_1 + \Lambda_2 \Omega_2 + \Lambda_3 \Omega_3 + \Lambda_4 \Omega_4 \right. \\
& \left. + \frac{1}{n^2 - 1} \right) + n S_{32,2} \frac{R_2}{R_1} \ln \frac{R_2}{R_1} \cot^4 \alpha \left[\Lambda_1 \Omega_1 \left(\frac{R_2}{R_1} \right)^{\lambda_1} + \Lambda_2 \Omega_2 \left(\frac{R_2}{R_1} \right)^{-\lambda_1} + \Lambda_3 \Omega_3 \left(\frac{R_2}{R_1} \right)^{\lambda_2} \right. \\
& \left. + \Lambda_4 \Omega_4 \left(\frac{R_2}{R_1} \right)^{-\lambda_2} \right]
\end{aligned}$$

(Equation continued on next page)

$$\begin{aligned}
& + \Lambda_4 \Omega_4 \left(\frac{R_2}{R_1} \right)^{-\lambda_2} + \frac{1}{n^2 - 1} \Big] + S_{41,2} \cot^3 \alpha \frac{B_\theta}{B_y} \left[\Omega_1 \psi_1 \left(\frac{R_2}{R_1} \right)^{\lambda_1} + \Omega_2 \psi_2 \left(\frac{R_2}{R_1} \right)^{-\lambda_1} + \Omega_3 \psi_3 \left(\frac{R_2}{R_1} \right)^{\lambda_2} \right. \\
& + \Omega_4 \psi_4 \left(\frac{R_2}{R_1} \right)^{-\lambda_2} + \frac{1}{(B_\theta/B_y)(1 - n^2)} \Big] + n S_{42,2} \cot^3 \alpha \left[\Lambda_1 \Omega_1 \left(\frac{R_2}{R_1} \right)^{\lambda_1} + \Lambda_2 \Omega_2 \left(\frac{R_2}{R_1} \right)^{-\lambda_1} \right. \\
& + \Lambda_3 \Omega_3 \left(\frac{R_2}{R_1} \right)^{\lambda_2} + \Lambda_4 \Omega_4 \left(\frac{R_2}{R_1} \right)^{-\lambda_2} + \frac{1}{n^2 - 1} \Big] + \cot^2 \alpha (S_{43,1} + S_{41,2}) \\
\gamma_5 = & \frac{R_2}{R_1} \left(\ln \frac{R_2}{R_1} \right)^2 \cot^3 \alpha S_{33,2} + S_{31,2} \frac{R_2}{R_1} \ln \frac{R_2}{R_1} \cot^4 \alpha \frac{B_\theta}{B_y} \left[\Gamma_1 \psi_1 \left(\frac{R_2}{R_1} \right)^{\lambda_1} + \Gamma_2 \psi_2 \left(\frac{R_2}{R_1} \right)^{-\lambda_1} \right. \\
& + \Gamma_3 \psi_3 \left(\frac{R_2}{R_1} \right)^{\lambda_2} + \Gamma_4 \psi_4 \left(\frac{R_2}{R_1} \right)^{-\lambda_2} - \frac{\ln R_2/R_1}{(B_\theta/B_y)(n^2 - 1)} + \frac{n^2 - \mu'_y}{P_2} \Big] + \cot \alpha \left[S_{44,1} + \left(\frac{R_2}{R_1} \right)^{-1} S_{44,2} \right] \\
& + \cot^3 \alpha \frac{B_\theta}{B_y} \left\{ S_{41,1} \left(\Gamma_1 \psi_1 + \Gamma_2 \psi_2 + \Gamma_3 \psi_3 + \Gamma_4 \psi_4 + \frac{n^2 - \frac{\mu'_\theta}{B_\theta/B_y}}{P_2} \right) + S_{41,2} \left[\Gamma_1 \psi_1 \left(\frac{R_2}{R_1} \right)^{\lambda_1} \right. \right. \\
& + \Gamma_2 \psi_2 \left(\frac{R_2}{R_1} \right)^{-\lambda_1} + \Gamma_3 \psi_3 \left(\frac{R_2}{R_1} \right)^{\lambda_2} + \Gamma_4 \psi_4 \left(\frac{R_2}{R_1} \right)^{-\lambda_2} - \frac{\ln R_2/R_1}{(B_\theta/B_y)(n^2 - 1)} + \frac{n^2 - \frac{\mu'_\theta}{B_\theta/B_y}}{P_2} \Big] \Big\} \\
& + n \cot^3 \alpha \left\{ S_{42,1} \left[\Lambda_1 \Gamma_1 + \Lambda_2 \Gamma_2 + \Lambda_3 \Gamma_3 + \Lambda_4 \Gamma_4 - \frac{1 - \frac{\mu'_\theta}{B_\theta/B_y}}{(n^2 - 1)^2} \right] + S_{42,2} \left[\Lambda_1 \Gamma_1 \left(\frac{R_2}{R_1} \right)^{\lambda_1} \right. \right.
\end{aligned}$$

(Equation continued on next page)

$$\begin{aligned}
& + \Lambda_2 \Gamma_2 \left(\frac{R_2}{R_1} \right)^{-\lambda_1} + \Lambda_3 \Gamma_3 \left(\frac{R_2}{R_1} \right)^{\lambda_2} + \Lambda_4 \Gamma_4 \left(\frac{R_2}{R_1} \right)^{-\lambda_2} + \frac{\ln R_2/R_1}{n^2 - 1} - \frac{1 - \frac{\mu'_\theta}{B_\theta/B_y}}{(n^2 - 1)^2} \Bigg] \Bigg\} \\
& + n S_{32,2} \cot \alpha \left[\Lambda_1 \Gamma_1 \left(\frac{R_2}{R_1} \right)^{\lambda_1} + \Lambda_2 \Gamma_2 \left(\frac{R_2}{R_1} \right)^{-\lambda_1} + \Lambda_3 \Gamma_3 \left(\frac{R_2}{R_1} \right)^{\lambda_2} + \Lambda_4 \Gamma_4 \left(\frac{R_2}{R_1} \right)^{-\lambda_2} + \frac{\ln R_2/R_1}{n^2 - 1} \right. \\
& \left. - \frac{1 - \frac{\mu'_\theta}{B_\theta/B_y}}{(n^2 - 1)^2} \right] + \ln \frac{R_2}{R_1} \cot^2 \alpha (S_{34,2} + S_{43,2}) \\
\gamma_{6m} = & S_{31,1} \frac{R_2}{R_1} \ln \frac{R_2}{R_1} \cot^4 \alpha \frac{B_\theta}{B_y} \left(\Phi_{1m} \psi_1 \left(\frac{R_2}{R_1} \right)^{\lambda_1} + \Phi_{2m} \psi_2 \left(\frac{R_2}{R_1} \right)^{-\lambda_1} + \Phi_{3m} \psi_3 \left(\frac{R_2}{R_1} \right)^{\lambda_2} \right. \\
& \left. + \Phi_{4m} \psi_4 \left(\frac{R_2}{R_1} \right)^{-\lambda_2} - \frac{(-1)^m \frac{m\pi}{\ln R_2/R_1} \left\{ \mu'_y \left[\left(\frac{m\pi}{\ln R_2/R_1} \right)^2 + 1 \right] - n^2 \right\}}{\left(\frac{m\pi}{\ln R_2/R_1} \right)^4 - P_1 \left(\frac{m\pi}{\ln R_2/R_1} \right)^2 + P_2} \right) \\
& + \frac{m\pi}{\ln R_2/R_1} \cot \alpha \left[S_{44,1} + (-1)^m \left(\frac{R_2}{R_1} \right)^{-1} S_{44,2} \right] + \cot^3 \alpha \frac{B_\theta}{B_y} S_{41,1} \left(\Phi_{1m} \psi_1 + \Phi_{2m} \psi_2 \right.
\end{aligned}$$

(Equation continued on next page)

$$\begin{aligned}
& + \Phi_{3m} \psi_3 + \Phi_{4m} \psi_4 - \frac{\frac{m\pi}{\ln R_2/R_1} \left\{ \frac{\mu'_\theta}{B_\theta/B_y} \left[\left(\frac{m\pi}{\ln R_2/R_1} \right)^2 + 1 \right] - n^2 \right\}}{\left(\frac{m\pi}{\ln R_2/R_1} \right)^4 - P_1 \left(\frac{m\pi}{\ln R_2/R_1} \right)^2 + P_2} \right) \\
& + S_{41,2} \left(\Phi_{1m} \psi_1 \left(\frac{R_2}{R_1} \right)^{\lambda_1} + \Phi_{2m} \psi_2 \left(\frac{R_2}{R_1} \right)^{-\lambda_1} + \Phi_{3m} \psi_3 \left(\frac{R_2}{R_1} \right)^{\lambda_2} + \Phi_{4m} \psi_4 \left(\frac{R_2}{R_1} \right)^{-\lambda_2} \right. \\
& \left. - \frac{(-1)^m \frac{m\pi}{\ln R_2/R_1} \left\{ \frac{\mu'_\theta}{B_\theta/B_y} \left[\left(\frac{m\pi}{\ln R_2/R_1} \right)^2 + 1 \right] - n^2 \right\}}{\left(\frac{m\pi}{\ln R_2/R_1} \right)^4 - P_1 \left(\frac{m\pi}{\ln R_2/R_1} \right)^2 + P_2} \right) + n \cot^3 \alpha \left\{ S_{42,1} \left[\Lambda_1 \Phi_{1m} + \Lambda_2 \Phi_{2m} \right. \right. \\
& \left. \left. + \Lambda_3 \Phi_{3m} + \Lambda_4 \Phi_{4m} - \frac{\frac{m\pi}{\ln R_2/R_1} \left(\frac{B_\theta}{B_y} - \mu'_\theta \right)}{\left(\frac{m\pi}{\ln R_2/R_1} \right)^4 - P_1 \left(\frac{m\pi}{\ln R_2/R_1} \right)^2 + P_2} \right] + S_{42,2} \left[\Lambda_1 \Phi_{1m} \left(\frac{R_2}{R_1} \right)^{\lambda_1} \right. \right. \\
& \left. \left. + \Lambda_2 \Phi_{2m} \left(\frac{R_2}{R_1} \right)^{-\lambda_1} + \Lambda_3 \Phi_{3m} \left(\frac{R_2}{R_1} \right)^{\lambda_2} + \Lambda_4 \Phi_{4m} \left(\frac{R_2}{R_1} \right)^{-\lambda_2} \right. \right. \\
& \left. \left. - \frac{(-1)^m \frac{m\pi}{\ln R_2/R_1} \left(\frac{B_\theta}{B_y} - \mu'_\theta \right)}{\left(\frac{m\pi}{\ln R_2/R_1} \right)^4 - P_1 \left(\frac{m\pi}{\ln R_2/R_1} \right)^2 + P_2} \right] \right\} + n \frac{R_2}{R_1} \ln \frac{R_2}{R_1} S_{32,2} \cot^4 \alpha \left[\Lambda_1 \Phi_{1m} \left(\frac{R_2}{R_1} \right)^{\lambda_1} \right.
\end{aligned}$$

(Equation continued on next page)

$$+ \Lambda_2 \Phi_{2m} \left(\frac{R_2}{R_1} \right)^{-\lambda_1} + \Lambda_3 \Phi_{3m} \left(\frac{R_2}{R_1} \right)^{\lambda_2} + \Lambda_4 \Phi_{4m} \left(\frac{R_2}{R_1} \right)^{-\lambda_2}$$

$$- \frac{(-1)^m \left(\frac{B_\theta}{B_y} - \mu'_\theta \right)}{\left(\frac{m\pi}{\ln R_2/R_1} \right)^4 - P_1 \left(\frac{m\pi}{\ln R_2/R_1} \right)^2 + P_2} \Bigg] + S_{34,2} \cot^2 \alpha \frac{m\pi}{\ln R_2/R_1} (-1)^m \ln \frac{R_2}{R_1}$$

$$\gamma_{7p} = \frac{p\pi}{x_1} \left(S_{41,1} \cot^3 \alpha \frac{B_\theta}{B_y} \left[\Omega_1 \psi_1 + \Omega_2 \psi_2 + \Omega_3 \psi_3 + \Omega_4 \psi_4 + \frac{1}{(B_\theta/B_y)(1-n^2)} \right] \right.$$

$$+ n S_{42,1} \cot^3 \alpha \left(\Lambda_1 \Omega_1 + \Lambda_2 \Omega_2 + \Lambda_3 \Omega_3 + \Lambda_4 \Omega_4 + \frac{1}{n^2 - 1} \right) + S_{43,1} \cot^2 \alpha$$

$$+ (-1)^p \left\{ S_{41,2} \cot^3 \alpha \frac{B_\theta}{B_y} \left[\Omega_1 \psi_1 \left(\frac{R_2}{R_1} \right)^{\lambda_1} + \Omega_2 \psi_2 \left(\frac{R_2}{R_1} \right)^{-\lambda_1} + \Omega_3 \psi_3 \left(\frac{R_2}{R_1} \right)^{\lambda_2} + \Omega_4 \psi_4 \left(\frac{R_2}{R_1} \right)^{-\lambda_2} \right. \right.$$

$$+ \frac{1}{(B_\theta/B_y)(1-n^2)} \Bigg] + n S_{42,2} \cot^3 \alpha \left[\Lambda_1 \Omega_1 \left(\frac{R_2}{R_1} \right)^{\lambda_1} + \Lambda_2 \Omega_2 \left(\frac{R_2}{R_1} \right)^{-\lambda_1} + \Lambda_3 \Omega_3 \left(\frac{R_2}{R_1} \right)^{\lambda_2} \right.$$

$$\left. + \Lambda_4 \Omega_4 \left(\frac{R_2}{R_1} \right)^{-\lambda_2} + \frac{1}{n^2 - 1} \right] + S_{43,2} \cot^2 \alpha \Bigg\}$$

$$\begin{aligned}
\gamma_{8p} = & \frac{p\pi}{x_1} \left(S_{41,1} \cot^3 \alpha \frac{B_\theta}{B_y} \left(\Gamma_1 \psi_1 + \Gamma_2 \psi_2 + \Gamma_3 \psi_3 + \Gamma_4 \psi_4 + \frac{n^2 - \frac{\mu'_\theta}{B_\theta/B_y}}{P_2} \right) \right. \\
& + n S_{42,1} \cot^3 \alpha \left[\Lambda_1 \Gamma_1 + \Lambda_2 \Gamma_2 + \Lambda_3 \Gamma_3 + \Lambda_4 \Gamma_4 - \frac{1 - \frac{\mu'_\theta}{B_\theta/B_y}}{(n^2 - 1)^2} \right] + S_{44,1} \cot \alpha \\
& + (-1)^p \left\{ S_{41,2} \cot^3 \alpha \frac{B_\theta}{B_y} \left[\Gamma_1 \psi_1 \left(\frac{R_2}{R_1} \right)^{\lambda_1} + \Gamma_2 \psi_2 \left(\frac{R_2}{R_1} \right)^{-\lambda_1} + \Gamma_3 \psi_3 \left(\frac{R_2}{R_1} \right)^{\lambda_2} + \Gamma_4 \psi_4 \left(\frac{R_2}{R_1} \right)^{-\lambda_2} \right. \right. \\
& - \frac{\ln R_2/R_1}{(B_\theta/B_y)(n^2 - 1)} + \frac{n^2 - \frac{\mu'_\theta}{B_\theta/B_y}}{P_2} \left. \right] + n S_{42,2} \cot^3 \alpha \left[\Lambda_1 \Gamma_1 \left(\frac{R_2}{R_1} \right)^{\lambda_1} + \Lambda_2 \Gamma_2 \left(\frac{R_2}{R_1} \right)^{-\lambda_1} \right. \\
& + \Lambda_3 \Gamma_3 \left(\frac{R_2}{R_1} \right)^{\lambda_2} + \Lambda_4 \Gamma_4 \left(\frac{R_2}{R_1} \right)^{-\lambda_2} + \frac{\ln R_2/R_1}{n^2 - 1} - \frac{1 - \frac{\mu'_\theta}{B_\theta/B_y}}{(n^2 - 1)^2} \left. \right] + S_{43,2} \cot^2 \alpha \ln \frac{R_2}{R_1} \\
& \left. + \left(\frac{R_2}{R_1} \right)^{-1} S_{44,2} \cot \alpha \right\}
\end{aligned}$$

$$\begin{aligned}
\gamma_{9mp} = & \frac{p\pi}{x_1} \left\{ S_{41,1} \cot^3 \alpha \frac{B_\theta}{B_y} \left(\Phi_{1m} \psi_1 + \Phi_{2m} \psi_2 + \Phi_{3m} \psi_3 + \Phi_{4m} \psi_4 \right. \right. \\
& - \frac{\frac{m\pi}{\ln R_2/R_1} \left\{ \frac{\mu'_\theta}{B_\theta/B_y} \left[\left(\frac{m\pi}{\ln R_2/R_1} \right)^2 + 1 \right] - n^2 \right\}}{\left(\frac{m\pi}{\ln R_2/R_1} \right)^4 - P_1 \left(\frac{m\pi}{\ln R_2/R_1} \right)^2 + P_2} \left. + nS_{42,1} \cot^3 \alpha \left[\Lambda_1 \Phi_{1m} + \Lambda_2 \Phi_{2m} \right. \right. \\
& + \Lambda_3 \Phi_{3m} + \Lambda_4 \Phi_{4m} - \frac{\frac{m\pi}{\ln R_2/R_1} \left(\frac{B_\theta}{B_y} - \mu'_\theta \right)}{\left(\frac{m\pi}{\ln R_2/R_1} \right)^4 - P_1 \left(\frac{m\pi}{\ln R_2/R_1} \right)^2 + P_2} \left. + S_{44,1} \cot \alpha \frac{m\pi}{\ln R_2/R_1} \right. \\
& + (-1)^p \left[S_{41,2} \cot^3 \alpha \frac{B_\theta}{B_y} \left(\psi_1 \Phi_{1m} \left(\frac{R_2}{R_1} \right)^{\lambda_1} + \psi_2 \Phi_{2m} \left(\frac{R_2}{R_1} \right)^{-\lambda_1} + \psi_3 \Phi_{3m} \left(\frac{R_2}{R_1} \right)^{\lambda_2} \right. \right. \\
& + \psi_4 \Phi_{4m} \left(\frac{R_2}{R_1} \right)^{-\lambda_2} - \frac{(-1)^m \frac{m\pi}{\ln R_2/R_1} \left\{ \frac{\mu'_\theta}{B_\theta/B_y} \left[\left(\frac{m\pi}{\ln R_2/R_1} \right)^2 + 1 \right] - n^2 \right\}}{\left(\frac{m\pi}{\ln R_2/R_1} \right)^4 - P_1 \left(\frac{m\pi}{\ln R_2/R_1} \right)^2 + P_2} \left. \right) \left. \right] \\
& + nS_{42,2} \cot^3 \alpha \left[\Lambda_1 \Phi_{1m} \left(\frac{R_2}{R_1} \right)^{\lambda_1} + \Lambda_2 \Phi_{2m} \left(\frac{R_2}{R_1} \right)^{-\lambda_1} + \Lambda_3 \Phi_{3m} \left(\frac{R_2}{R_1} \right)^{\lambda_2} + \Lambda_4 \Phi_{4m} \left(\frac{R_2}{R_1} \right)^{-\lambda_2} \right]
\end{aligned}$$

(Equation continued on next page)

$$\left. - \frac{(-1)^m \frac{m\pi}{\ln R_2/R_1} \left(\frac{B_\theta}{B_y} - \mu'_\theta \right)}{\left(\frac{m\pi}{\ln R_2/R_1} \right)^4 - P_1 \left(\frac{m\pi}{\ln R_2/R_1} \right)^2 + P_2} + \left(\frac{R_2}{R_1} \right)^{-1} S_{44,2} \cot \alpha \frac{m\pi}{\ln R_2/R_1} (-1)^{p+m} \right\}$$

$$\gamma_{9p} = \gamma_{9mp} \quad \text{with } m = p$$

The quantities β are defined as $\beta = \gamma$ with S_{ij} replaced by $M_{ij} \sin^4 \alpha$.

For symmetric deformations, the quantities t_i are replaced by t_{oi} defined as

$$t_{o1} = \Omega_{o1} \left[\left(\frac{R_2}{R_1} \right)^{\lambda_o} - 1 \right] + \Omega_{o2} \left[\left(\frac{R_2}{R_1} \right)^{-\lambda_o} - 1 \right]$$

$$t_{o2} = t_{o1}$$

with Ω_{oi} replaced by Γ_{oi} .

$$t_{o3} = t_{o1}$$

with Ω_{oi} replaced by Φ_{oim} and the additional term

$$- \frac{\frac{m\pi}{x_1} [1 - (-1)^m]}{\left(\frac{m\pi}{x_1} \right)^2 + (B_\theta/B_y)}$$

$$t_{o4} = \frac{\Omega_{o1}}{\lambda_o} \left[1 + \left(\frac{R_2}{R_1} \right)^{\lambda_o} \left(\lambda_o \ln \frac{R_2}{R_1} - 1 \right) \right] - \frac{\Omega_{o2}}{\lambda_o} \left[1 - \left(\frac{R_2}{R_1} \right)^{-\lambda_o} \left(\lambda_o \ln \frac{R_2}{R_1} + 1 \right) \right]$$

$$t_{o5} = t_{o4}$$

with Ω_{oi} replaced by Γ_{oi} .

$$t_{o6m} = t_{o4}$$

with Ω_{oi} replaced by Φ_{oim} and the additional term

$$+ \frac{\frac{m\pi}{x_1} (-1)^m \ln R_2/R_1}{\left(\frac{m\pi}{x_1}\right)^2 + (B_\theta/B_y)}$$

$$t_{o7p} = \frac{\lambda_o \frac{p\pi}{x_1}}{\left(\frac{p\pi}{x_1}\right)^2 + (B_\theta/B_y)} \left\{ \Omega_{o1} \left[1 - (-1)^p \left(\frac{R_2}{R_1}\right)^{\lambda_o} \right] - \Omega_{o2} \left[1 - (-1)^p \left(\frac{R_2}{R_1}\right)^{-\lambda_o} \right] \right\}$$

$$t_{o8p} = t_{o7p}$$

with Ω_{oi} replaced by Γ_{oi} .

$$t_{o9mp} = t_{o7p}$$

with Ω_{oi} replaced by Φ_{oim} .

$$t_{o9p} = t_{o7p}$$

with Ω_{oi} replaced by Φ_{oim} and the additional term

$$- \frac{\frac{1}{2} \left(\frac{p\pi}{x_1}\right)^2 \ln R_2/R_1}{\left(\frac{p\pi}{x_1}\right)^2 + (B_\theta/B_y)}$$

For symmetric deformation, the quantities γ_i are replaced by γ_{oi} defined as

$$\begin{aligned} \gamma_{o1} = & \cot^4 \alpha \left\{ S_{13,2} \left[\Omega_{o1} \left(\sqrt{B_\theta/B_y} - \mu'_\theta \right) - \Omega_{o2} \left(\sqrt{B_\theta/B_y} + \mu'_\theta \right) + 1 \right] \right. \\ & + \frac{R_2}{R_1} S_{13,2} \left[\Omega_{o1} \left(\frac{R_2}{R_1} \right)^{\lambda_o} \left(\sqrt{B_\theta/B_y} - \mu'_\theta \right) - \Omega_{o2} \left(\frac{R_2}{R_1} \right)^{-\lambda_o} \left(\sqrt{B_\theta/B_y} + \mu'_\theta \right) + 1 \right] \Big\} \\ & + \cot^3 \alpha \left(S_{33,1} + \frac{R_2}{R_1} S_{33,2} \right) \end{aligned}$$

$$\begin{aligned}
\gamma_{o2} = & \cot^4 \alpha \left\{ S_{13,1} \left[\Gamma_{o1} \left(\sqrt{B_\theta/B_y} - \mu'_\theta \right) - \Gamma_{o2} \left(\sqrt{B_\theta/B_y} + \mu'_\theta \right) - \mu'_y \right] \right. \\
& + \frac{R_2}{R_1} S_{13,2} \left[\Gamma_{o1} \left(\frac{R_2}{R_1} \right)^{\lambda_o} \left(\sqrt{B_\theta/B_y} - \mu'_\theta \right) - \Gamma_{o2} \left(\frac{R_2}{R_1} \right)^{-\lambda_o} \left(\sqrt{B_\theta/B_y} + \mu'_\theta \right) + \ln \frac{R_2}{R_1} - \mu'_y \right] \Big\} \\
& + \frac{R_2}{R_1} \ln \frac{R_2}{R_1} S_{33,2} \cot^3 \alpha + \cot^2 \alpha (S_{34,1} + S_{34,2})
\end{aligned}$$

$$\begin{aligned}
\gamma_{o3m} = & \cot^4 \alpha \left\{ \frac{R_2}{R_1} + S_{31,2} \left[\Phi_{o1m} \left(\sqrt{B_\theta/B_y} - \mu'_\theta \right) \left(\frac{R_2}{R_1} \right)^{\lambda_o} - \Phi_{o2m} \left(\sqrt{B_\theta/B_y} + \mu'_\theta \right) \left(\frac{R_2}{R_1} \right)^{-\lambda_o} \right. \right. \\
& - \frac{\mu'_\theta \frac{m\pi}{x_1} (-1)^m}{\left(\frac{m\pi}{x_1} \right)^2 + (B_\theta/B_y)} \Big] + S_{31,1} \left[\Phi_{o1m} \left(\sqrt{B_\theta/B_y} - \mu'_\theta \right) - \Phi_{o2m} \left(\sqrt{B_\theta/B_y} + \mu'_\theta \right) \right. \\
& \left. \left. - \frac{\mu'_\theta \frac{m\pi}{x_1}}{\left(\frac{m\pi}{x_1} \right)^2 + (B_\theta/B_y)} \right] \right\} + \frac{m\pi}{x_1} \cot^2 \alpha [S_{34,1} + (-1)^m S_{34,2}]
\end{aligned}$$

$$\begin{aligned}
\gamma_{o4} = & \cot^3 \alpha \left\{ S_{14,1} \left[\Omega_{o1} \left(\sqrt{B_\theta/B_y} - \mu'_\theta \right) - \Omega_{o2} \left(\sqrt{B_\theta/B_y} + \mu'_\theta \right) + 1 \right] \right. \\
& + S_{14,2} \left[\Omega_{o1} \left(\sqrt{B_\theta/B_y} - \mu'_\theta \right) \left(\frac{R_2}{R_1} \right)^{\lambda_o} - \Omega_{o2} \left(\sqrt{B_\theta/B_y} + \mu'_\theta \right) \left(\frac{R_2}{R_1} \right)^{-\lambda_o} + 1 \right] \Big\} \\
& + \cot^2 \alpha (S_{34,1} + S_{34,2}) + \frac{R_2}{R_1} \ln \frac{R_2}{R_1} \left\{ S_{13,2} \cot^4 \alpha \left[\Omega_{o1} \left(\sqrt{B_\theta/B_y} - \mu'_\theta \right) \left(\frac{R_2}{R_1} \right)^{\lambda_o} \right. \right. \\
& \left. \left. - \Omega_{o2} \left(\sqrt{B_\theta/B_y} + \mu'_\theta \right) \left(\frac{R_2}{R_1} \right)^{-\lambda_o} + 1 \right] + S_{33,2} \cot^3 \alpha \right\}
\end{aligned}$$

$$\begin{aligned}
\gamma_{o5} = & \frac{R_2}{R_1} \ln \frac{R_2}{R_1} \left\{ S_{13,2} \cot^4 \alpha \left[\Gamma_{o1} \left(\sqrt{B_\theta/B_y} - \mu'_\theta \right) \left(\frac{R_2}{R_1} \right)^{\lambda_o} - \Gamma_{o2} \left(\sqrt{B_\theta/B_y} + \mu'_\theta \right) \left(\frac{R_2}{R_1} \right)^{-\lambda_o} \right. \right. \\
& \left. \left. + \ln \frac{R_2}{R_1} - \frac{\mu'_\theta}{B_\theta/B_y} \right] + S_{33,2} \ln \frac{R_2}{R_1} \cot^3 \alpha \right\} + S_{34,2} \cot^2 \alpha \ln \frac{R_2}{R_1} \\
& + S_{41,2} \cot^3 \alpha \left[\Gamma_{o1} \left(\sqrt{B_\theta/B_y} - \mu'_\theta \right) \left(\frac{R_2}{R_1} \right)^{\lambda_o} - \Gamma_{o2} \left(\sqrt{B_\theta/B_y} + \mu'_\theta \right) \left(\frac{R_2}{R_1} \right)^{-\lambda_o} \right. \\
& \left. + \ln \frac{R_2}{R_1} - \frac{\mu'_\theta}{B_\theta/B_y} \right] + S_{43,2} \ln \frac{R_2}{R_1} \cot^2 \alpha + \left(\frac{R_2}{R_1} \right)^{-1} S_{44,2} \cot \alpha + S_{44,1} \cot \alpha \\
& + S_{41,1} \cot^3 \alpha \left[\Gamma_{o1} \left(\sqrt{B_\theta/B_y} - \mu'_\theta \right) - \Gamma_{o2} \left(\sqrt{B_\theta/B_y} + \mu'_\theta \right) - \frac{\mu'_\theta}{B_\theta/B_y} \right] \\
\gamma_{o6m} = & \frac{R_2}{R_1} \ln \frac{R_2}{R_1} \cot^4 \alpha S_{13,2} \left[\Phi_{o1m} \left(\sqrt{B_\theta/B_y} - \mu'_\theta \right) \left(\frac{R_2}{R_1} \right)^{\lambda_o} - \Phi_{o2m} \left(\sqrt{B_\theta/B_y} + \mu'_\theta \right) \left(\frac{R_2}{R_1} \right)^{-\lambda_o} \right. \\
& \left. - \frac{\mu'_\theta \frac{m\pi}{x_1} (-1)^m}{\left(\frac{m\pi}{x_1} \right)^2 + (B_\theta/B_y)} \right] + \ln \frac{R_2}{R_1} \cot^2 \alpha \frac{m\pi}{x_1} (-1)^m S_{34,2} + \cot^3 \alpha \left\{ S_{14,1} \Phi_{o1m} \right. \\
& \times \left[\left(\sqrt{B_\theta/B_y} - \mu'_\theta \right) - \Phi_{o2m} \left(\sqrt{B_\theta/B_y} + \mu'_\theta \right) - \frac{\mu'_\theta \frac{m\pi}{x_1}}{\left(\frac{m\pi}{x_1} \right)^2 + (B_\theta/B_y)} \right] + S_{14,2} \Phi_{o1m} \\
& \times \left[\left(\sqrt{B_\theta/B_y} - \mu'_\theta \right) \left(\frac{R_2}{R_1} \right)^{\lambda_o} - \Phi_{o2m} \left(\sqrt{B_\theta/B_y} + \mu'_\theta \right) \left(\frac{R_2}{R_1} \right)^{-\lambda_o} - \frac{\mu'_\theta \frac{m\pi}{x_1} (-1)^m}{\left(\frac{m\pi}{x_1} \right)^2 + (B_\theta/B_y)} \right] \Bigg\} \\
& + \frac{m\pi}{x_1} \cot \alpha \left[S_{44,1} + (-1)^m \left(\frac{R_2}{R_1} \right)^{-1} S_{44,2} \right]
\end{aligned}$$

$$\gamma_{o7p} = \frac{p\pi}{x_1} \left(\cot^3 \alpha \left\{ S_{14,1} \left[\Omega_{o1} \left(\sqrt{B_\theta/B_y} - \mu'_\theta \right) - \Omega_{o2} \left(\sqrt{B_\theta/B_y} + \mu'_\theta \right) + 1 \right] + (-1)^p S_{14,2} \right. \right. \\ \times \left. \left[\Omega_{o1} \left(\sqrt{B_\theta/B_y} - \mu'_\theta \right) \left(\frac{R_2}{R_1} \right)^{\lambda_o} - \Omega_{o2} \left(\sqrt{B_\theta/B_y} + \mu'_\theta \right) \left(\frac{R_2}{R_1} \right)^{-\lambda_o} + 1 \right] \right\} \\ \left. + \cot^2 \alpha \left[S_{34,1} + (-1)^p S_{34,2} \right] \right)$$

$$\gamma_{o8p} = \frac{p\pi}{x_1} \left(\cot^3 \alpha \left\{ S_{14,1} \left[\Gamma_{o1} \left(\sqrt{B_\theta/B_y} - \mu'_\theta \right) - \Gamma_{o2} \left(\sqrt{B_\theta/B_y} + \mu'_\theta \right) - \frac{\mu'_\theta}{B_\theta/B_y} \right] \right. \right. \\ + (-1)^p S_{14,2} \left[\Gamma_{o1} \left(\sqrt{B_\theta/B_y} - \mu'_\theta \right) \left(\frac{R_2}{R_1} \right)^{\lambda_o} - \Gamma_{o2} \left(\sqrt{B_\theta/B_y} + \mu'_\theta \right) \left(\frac{R_2}{R_1} \right)^{-\lambda_o} \right. \\ \left. \left. + \ln \frac{R_2}{R_1} - \frac{\mu'_\theta}{B_\theta/B_y} \right] \right\} + (-1)^p \ln \frac{R_2}{R_1} \cot^2 \alpha S_{34,2} + \cot \alpha \left[S_{44,1} + (-1)^p \left(\frac{R_2}{R_1} \right)^{-1} S_{44,2} \right] \right)$$

$$\gamma_{o9mp} = \frac{p\pi}{x_1} \cot^3 \alpha \left\{ S_{14,1} \left[\Phi_{o1m} \left(\sqrt{B_\theta/B_y} - \mu'_\theta \right) - \Phi_{o2m} \left(\sqrt{B_\theta/B_y} + \mu'_\theta \right) - \frac{\mu'_\theta \frac{m\pi}{x_1}}{\left(\frac{m\pi}{x_1} \right)^2 + (B_\theta/B_y)} \right] \right. \\ + (-1)^p S_{14,2} \left[\Phi_{o1m} \left(\sqrt{B_\theta/B_y} - \mu'_\theta \right) \left(\frac{R_2}{R_1} \right)^{\lambda_o} - \Phi_{o2m} \left(\sqrt{B_\theta/B_y} + \mu'_\theta \right) \left(\frac{R_2}{R_1} \right)^{-\lambda_o} \right. \\ \left. \left. - \frac{\mu'_\theta \frac{m\pi}{x_1} (-1)^m}{\left(\frac{m\pi}{x_1} \right)^2 + (B_\theta/B_y)} \right] + \frac{p\pi}{x_1} \frac{m\pi}{x_1} \cot \alpha \left[S_{44,1} + (-1)^{m+p} \left(\frac{R_2}{R_1} \right)^{-1} S_{44,2} \right] \right\}$$

$$\begin{aligned}
\gamma_{o9p} = \frac{m\pi}{x_1} \cot^3 \alpha \left\{ S_{14,1} \left[\Phi_{o1m} \left(\sqrt{B_\theta/B_y} - \mu'_\theta \right) - \Phi_{o2m} \left(\sqrt{B_\theta/B_y} + \mu'_\theta \right) - \frac{\mu'_\theta \frac{m\pi}{x_1}}{\left(\frac{m\pi}{x_1} \right)^2 + (B_\theta/B_y)} \right] \right. \\
+ (-1)^m S_{14,2} \left[\Phi_{o1m} \left(\sqrt{B_\theta/B_y} - \mu'_\theta \right) \left(\frac{R_2}{R_1} \right)^{\lambda_o} - \Phi_{o2m} \left(\sqrt{B_\theta/B_y} + \mu'_\theta \right) \left(\frac{R_2}{R_1} \right)^{-\lambda_o} \right. \\
\left. \left. - \frac{\mu'_\theta \frac{m\pi}{x_1}}{\left(\frac{m\pi}{x_1} \right)^2 + (B_\theta/B_y)} \right] \right\} + \left(\frac{m\pi}{x_1} \right)^2 \cot \alpha \left[S_{44,1} + \left(\frac{R_2}{R_1} \right)^{-1} S_{44,2} \right]
\end{aligned}$$

For symmetric deformations, the quantities β_i are replaced by β_{oi} defined as

$$\beta_{oi} = \gamma_{oi}$$

with S_{ij} replaced by $M_{ij} \sin^4 \alpha$.

EXPRESSIONS FOR S_{ij} AND M_{ij}

The expressions for S_{ij} are

$$S_{11} = \bar{S}_{11} \cos^2 \alpha + 2\bar{S}_{13} \sin \alpha \cos \alpha + \bar{S}_{33} \sin^2 \alpha$$

$$S_{12} = S_{21} = \bar{S}_{12} \cos \alpha + \bar{S}_{23} \sin \alpha$$

$$S_{13} = S_{31} = (\sin^2 \alpha - \cos^2 \alpha) \bar{S}_{13} + (\bar{S}_{11} - \bar{S}_{33}) \sin \alpha \cos \alpha$$

$$S_{14} = S_{41} = \bar{S}_{14} \cos \alpha + \bar{S}_{34} \sin \alpha$$

$$S_{22} = \bar{S}_{22}$$

$$S_{23} = S_{32} = -\bar{S}_{23} \cos \alpha + \bar{S}_{12} \sin \alpha$$

$$S_{24} = S_{42} = \bar{S}_{24}$$

$$S_{33} = \bar{S}_{11} \sin^2 \alpha + \bar{S}_{33} \cos^2 \alpha - 2\bar{S}_{13} \sin \alpha \cos \alpha$$

$$S_{34} = S_{43} = -\bar{S}_{34} \cos \alpha + \bar{S}_{14} \sin \alpha$$

$$S_{44} = \bar{S}_{44}$$

$$\begin{aligned} \bar{S}_{11} = \frac{\eta^2 (R_1/R)^2 Z \bar{A}}{\bar{z}_0} & \left\{ \frac{(R_1/R)^2}{\bar{z}_0^2} \left[\frac{\eta^2 \bar{I}_z}{\cos \alpha} + \frac{\bar{J}}{2(1 + \mu_r) \cos \alpha} - 2\eta^2 \frac{z_0}{R} \tan \alpha \bar{I}_{yz} \right. \right. \\ & \left. \left. + \eta^2 \left(\frac{z_0}{R} \right)^2 \tan \alpha \sin \alpha \bar{I}_y \right] + \eta^2 \left(\frac{z_0}{R} \right)^2 \tan \alpha \sin \alpha \right\} \end{aligned}$$

$$\bar{S}_{12} = \eta^3 \left(\frac{R_1}{R} \right)^2 Z \bar{A} \left[\frac{(R_1/R)^2}{\bar{z}_0^2 \cos \alpha} \left(\bar{I}_{yz} - \frac{z_0}{R} \sin \alpha \bar{I}_y \right) - \frac{z_0}{R} \tan \alpha \right]$$

$$\bar{S}_{13} = \frac{\eta^2 (R_1/R)^2 Z \bar{A}}{\bar{z}_0} \left[\eta^2 \frac{(R_1/R)^2}{\bar{z}_0} \left(\frac{\bar{I}_{yz}}{\cos \alpha} - \tan \alpha \frac{z_0}{R} \bar{I}_y \right) - \tan \alpha \frac{z_0}{R} \left(1 + \eta^2 \frac{z_0}{R} \cos \alpha \right) \right]$$

$$\begin{aligned}
\bar{S}_{14} &= \frac{\eta^2 (R_1/R)^3 Z\bar{A}}{\bar{z}_0^2} \left\{ \left(\eta^2 \frac{z_0}{R} \frac{\cos \alpha}{\bar{z}_0} - 1 \right) \bar{I}_z + \frac{1}{2(1+\mu_r)} \left(\frac{z_0}{R} \frac{\cos \alpha}{\bar{z}_0} - 1 \right) \bar{J} \right. \\
&\quad \left. + \frac{z_0}{R} \sin \alpha \left[\frac{\eta^2}{\bar{z}_0} \left(1 - \frac{z_0}{R} \cos \alpha \right) + 1 \right] \bar{I}_{yz} - \eta^2 \left(\frac{z_0}{R} \right)^2 \frac{\sin^2 \alpha}{\bar{z}_0} \bar{I}_y \right\} - \eta^2 \left(\frac{z_0}{R} \right)^2 \frac{R_1}{R} \frac{\sin^2 \alpha}{\bar{z}_0} Z\bar{A} \\
\bar{S}_{22} &= \frac{\eta^2 \bar{z}_0 (R_1/R)^2 Z\bar{A}}{\cos \alpha} \left[1 + \frac{(R_1/R)^2}{\bar{z}_0^2} \bar{I}_y \right] \\
\bar{S}_{23} &= \frac{\eta (R_1/R)^2 Z\bar{A}}{\cos \alpha} \left[\eta^2 \frac{(R_1/R)^2}{\bar{z}_0} \bar{I}_y + 1 + \eta^2 \frac{z_0}{R} \cos \alpha \right] \\
\bar{S}_{24} &= \eta Z\bar{A} \left[\left(\frac{R_1}{R} \right)^3 \frac{1}{\bar{z}_0} \left(\frac{\eta^2}{\bar{z}_0} \frac{z_0}{R} \cos \alpha - 1 \right) \bar{I}_{yz} + \frac{z_0}{R} \sin \alpha \frac{R_1}{R} + \eta^2 \frac{z_0}{R} \frac{\sin \alpha}{\bar{z}_0^2} \left(\frac{R_1}{R} \right)^3 \bar{I}_y \right] \\
\bar{S}_{33} &= \frac{(R_1/R)^2 Z\bar{A}}{\bar{z}_0 \cos \alpha} \left[\left(1 + \eta^2 \cos \alpha \frac{z_0}{R} \right)^2 + \eta^4 \left(\frac{R_1}{R} \right)^2 \bar{I}_y \right] \\
\bar{S}_{34} &= \frac{\eta^2 (R_1/R)^3 Z\bar{A}}{\bar{z}_0} \left[\left(\frac{\eta^2}{\bar{z}_0} \frac{z_0}{R} \cos \alpha - 1 \right) \bar{I}_{yz} + \eta^2 \frac{z_0}{R} \frac{\sin \alpha}{\bar{z}_0} \bar{I}_y \right] + \frac{\sin \alpha}{\bar{z}_0} \frac{z_0}{R} \frac{R_1}{R} Z\bar{A} \left(1 + \eta^2 \frac{z_0}{R} \cos \alpha \right) \\
\bar{S}_{44} &= \frac{\cos \alpha Z\bar{A}}{\bar{z}_0} \left\{ \left(\frac{R_1}{R} \right)^2 \left[\left(1 - \frac{\eta^2}{\bar{z}_0} \frac{z_0}{R} \cos \alpha \right)^2 \bar{I}_z + \frac{2\eta^2}{\bar{z}_0} \frac{z_0}{R} \sin \alpha \left(\frac{\eta^2}{\bar{z}_0} \frac{z_0}{R} \cos \alpha - 1 \right) \bar{I}_{yz} \right. \right. \\
&\quad \left. \left. + \frac{\eta^2 \left(1 - \frac{1}{\bar{z}_0} \frac{z_0}{R} \cos \alpha \right)^2}{2(1+\mu_r)} \bar{J} + \eta^4 \left(\frac{z_0}{R} \right)^2 \frac{\sin^2 \alpha}{\bar{z}_0^2} \bar{I}_y \right] + \left(\frac{z_0}{R} \right)^2 \sin^2 \alpha \right\}
\end{aligned}$$

where z_0 is the eccentricity of ring centroidal axis measured from inside shell surface, positive for external ring, and

$$\bar{A} = \frac{E_r A_r}{R_1 B_\theta}$$

$$\bar{I}_y = \frac{I_z \cos^2 \alpha + I_y \sin^2 \alpha + 2I_{yz} \sin \alpha \cos \alpha}{R_1^2 A_r}$$

$$\bar{I}_z = \frac{I_y \cos^2 \alpha + I_z \sin^2 \alpha - 2I_{yz} \sin \alpha \cos \alpha}{R_1^2 A_r}$$

$$\bar{I}_{yz} = \frac{I_{yz}(\sin^2 \alpha - \cos^2 \alpha) + \sin \alpha \cos \alpha (I_z - I_y)}{R_1^2 A_r}$$

$$\bar{J} = \frac{J}{R_1^2 A_r}$$

$$\bar{z}_0 = 1 + \frac{z_0}{R} \cos \alpha$$

The expressions for M_{ij} are

$$M_{11} = \bar{M}_{11} \cos^2 \alpha + 2\bar{M}_{13} \sin \alpha \cos \alpha + \bar{M}_{33} \sin^2 \alpha$$

$$M_{12} = M_{21} = \bar{M}_{12} \cos \alpha + \bar{M}_{23} \sin \alpha$$

$$M_{13} = M_{31} = (\sin^2 \alpha - \cos^2 \alpha) \bar{M}_{13} + (\bar{M}_{11} - \bar{M}_{33}) \sin \alpha \cos \alpha$$

$$M_{14} = M_{41} = \bar{M}_{14} \cos \alpha + \bar{M}_{34} \sin \alpha$$

$$M_{22} = \bar{M}_{22}$$

$$M_{23} = M_{32} = -\bar{M}_{23} \cos \alpha + \bar{M}_{12} \sin \alpha$$

$$M_{24} = M_{42} = \bar{M}_{24}$$

$$M_{33} = \bar{M}_{11} \sin^2 \alpha + \bar{M}_{33} \cos^2 \alpha - 2\bar{M}_{13} \sin \alpha \cos \alpha$$

$$M_{34} = M_{43} = -\bar{M}_{34} \cos \alpha + \bar{M}_{14} \sin \alpha$$

$$M_{44} = \bar{M}_{44}$$

where

$$\begin{aligned} \bar{M}_{11} = \bar{\rho} \cos \alpha \left\{ \bar{z}_0 \left[1 + \eta^2 \left(\frac{z_0}{R} \right)^2 \sin^2 \alpha \right] + \eta^2 \left(\frac{z_0}{R} \right)^2 \left(\frac{R_1}{R} \right)^2 \frac{\sin^2 \alpha}{\bar{z}_0} \bar{I}_y + \eta^2 \left(\frac{R_1}{R} \right)^2 \frac{\bar{I}_z}{\bar{z}_0} \right. \\ \left. - 2\eta^2 \frac{z_0}{R} \frac{\sin \alpha}{\bar{z}_0} \left(\frac{R_1}{R} \right)^2 \bar{I}_{yz} \right\} \end{aligned}$$

$$\bar{M}_{12} = \bar{\rho} \cos \alpha \left[-\eta \left(\frac{z_0}{R} \right) \sin \alpha \bar{z}_0^2 - \eta \frac{z_0}{R} \left(\frac{R_1}{R} \right)^2 \sin \alpha \bar{I}_y + \eta \left(\frac{R_1}{R} \right)^2 \bar{I}_{yz} \right]$$

$$\bar{M}_{13} = \bar{\rho} \cos \alpha \left[-\eta^2 \left(\frac{z_0}{R} \right)^2 \bar{z}_0 \sin \alpha \cos \alpha - \eta^2 \left(\frac{z_0}{R} \right) \sin \alpha \left(\frac{R_1}{R} \right)^2 \bar{I}_y + \eta^2 \left(\frac{R_1}{R} \right)^2 \bar{I}_{yz} \right]$$

$$\begin{aligned} \bar{M}_{14} = \bar{\rho} \cos^2 \alpha \left[\left(\frac{R_1}{R} \right)^{-1} \left(\frac{z_0}{R} \right) \cos \alpha \bar{z}_0 - \eta^2 \frac{R_1}{R} \left(\frac{z_0}{R} \right)^2 \frac{\sin^2 \alpha}{\bar{z}_0} \bar{I}_y + \eta^2 \frac{\cos \alpha}{\bar{z}_0} \left(\frac{z_0}{R} \right) \frac{R_1}{R} \bar{I}_z \right. \\ \left. + \eta^2 \left(\frac{z_0}{R} \right) \frac{R_1}{R} \frac{\sin \alpha}{\bar{z}_0} \left(1 - \frac{z_0}{R} \cos \alpha \right) \bar{I}_{yz} \right] \end{aligned}$$

$$\bar{M}_{22} = \bar{\rho} \bar{z}_0 \cos \alpha \left[\bar{z}_0^2 + \left(\frac{R_1}{R} \right)^2 \bar{I}_y \right]$$

$$\bar{M}_{23} = \eta \bar{\rho} \bar{z}_0 \cos \alpha \left[\frac{z_0}{R} \bar{z}_0 \cos \alpha + \left(\frac{R_1}{R} \right)^2 \bar{I}_y \right]$$

$$\bar{M}_{24} = \eta \bar{\rho} \frac{z_0}{R} \cos^2 \alpha \frac{R_1}{R} \left(\bar{I}_y \sin \alpha + \bar{I}_{yz} \cos \alpha \right)$$

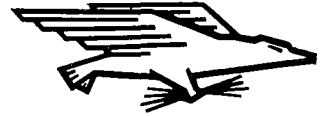
$$\bar{M}_{33} = \bar{\rho} \cos \alpha \bar{z}_0 \left[1 + \eta^2 \left(\frac{z_0}{R} \right)^2 \cos^2 \alpha + \eta^2 \left(\frac{R_1}{R} \right)^2 \bar{I}_y \right]$$

$$\bar{M}_{34} = \bar{\rho} \cos^2 \alpha \left(\frac{R_1}{R} \right)^{-1} \left(\frac{z_0}{R} \right) \left[\sin \alpha \bar{z}_0 + \eta^2 \sin \alpha \left(\frac{R_1}{R} \right)^2 \bar{I}_y + \eta^2 \cos \alpha \left(\frac{R_1}{R} \right)^2 \bar{I}_{yz} \right]$$

$$\begin{aligned} \bar{M}_{44} = & \bar{\rho} \left(\frac{R_1}{R} \right)^{-2} \cos^3 \alpha \left(\bar{z}_0 \left(\frac{z_0}{R} \right)^2 + \frac{(R_1/R)^2}{\bar{z}_0} \left\{ \bar{I}_y \left[\bar{z}_0^2 + \eta^2 \left(\frac{z_0}{R} \right)^2 \sin^2 \alpha \right] \right. \right. \\ & \left. \left. + \bar{I}_z \left[\bar{z}_0^2 + \eta^2 \left(\frac{z_0}{R} \right)^2 \cos^2 \alpha \right] + 2 \left(\frac{z_0}{R} \right)^2 \sin \alpha \cos \alpha \eta^2 \bar{I}_{yz} \right\} \right) \end{aligned}$$

$$\bar{\rho} = \frac{\rho A_r}{\gamma R_1}$$

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